Dynamic Matching and Bargaining: The Role of Private Deadlines.*

Sjaak Hurkens
Institut d'Anàlisi Econòmica (CSIC) and CREA

Nir Vulkan
Saïd Business School and Worcester College, University of Oxford

January 2007

Abstract

We consider a dynamic model where traders in each period are matched randomly into pairs who then bargain about the division of a fixed surplus. When agreement is reached the traders leave the market. Traders who do not come to an agreement return next period in which they will be matched again, as long as their deadline has not expired yet. New traders enter exogenously in each period. We assume that traders have private information about their deadline. We define and characterize the stationary equilibrium configurations. It is shown that the heterogeneity of deadlines may but need not cause delay and that deadlines may even be missed altogether. If there is no delay, all traders receive the same payoff. In equilibria with delay, traders with longer deadlines fare better than traders with short deadlines. We discuss ways of resolving the inefficiencies caused by delay and missed deadlines.

Key Words: Bargaining, deadlines, markets, private information.

JEL Classification Numbers: C73; C78

*Sjaak Hurkens gratefully acknowledges financial support through grant SEJ2006-01717 from the Spanish Ministry of Education and Science (MEC) and through grant CONSOLIDER-INGENIO 2010 (CSD2006-00016).

1 Introduction.

The driving force of any dynamic bargaining model is the assumption that people prefer to realize gains early rather than late. If bargaining partners do not care about the time of agreement, there is no incentive to come to agreements in the first place and bargaining could go on forever. The eagerness to reach early agreements has been modelled by making the (expected) bargaining surplus shrink over time. This can be done by introducing a discount factor strictly less than one, by assuming a positive probability of breakdown of negotiation, or by assuming that bargaining partners face a fixed cost of bargaining per period. All these models of time preferences assume stationarity (Rubinstein, 1982): agreement x today is preferred over agreement y tomorrow if and only if agreement x in period t is preferred over agreement y in period $t+1$. This assumption implies that, in the subgame perfect equilibrium of the infinite alternating offer game, it does not matter how much time has elapsed because the bargaining cost is sunk. In this paper, like in our companion paper Hurkens and Vulkan (2006), we want to consider deadlines as an alternative way to express a preference for early agreements in bargaining. In our interpretation, an agreement reached after the deadline has expired has no value. This violates the stationarity assumption.

We have various motives for studying deadlines. First, deadlines are present in many real bargaining situations and one would like to know how deadlines influence the bargaining strategies and outcomes. In particular, we will be interested in the case that the bargaining partners may have different deadlines and that their deadline may be private information. Second, it is often easier for a person to state by which date an agreement must be reached (say a month from now) than to make precise how much he is willing to pay extra to have an agreement today rather than tomorrow. This is particularly important with the growth of e-commerce, where customers and organizations typically interact using software agents (as is the case in eBay, where bidding is done through a proxy agent). These software agents must often be programmed with a deadline in order to ensure the termination of the protocol in which they take part. Finally, delegates who bargain on behalf of other people are often given a final date at which an agreement must have been reached.

This paper studies the interaction of large groups of buyers and sellers who arrive at an exogenous rate to the market. Buyers and sellers are randomly matched into pairs and bargaining takes place in each match about the division of the surplus. We assume that the size of the surplus is fixed in order to focus on the effect of heterogeneous deadlines. If an agreement is reached, the traders disappear with their gains from the market. If there is no agreement, and a trader's deadline has expired, the trader will disappear from the market with no surplus. In the case of disagreement and a non-expiring deadline, the trader returns next period in which he will again be matched, with a different partner. The deadline of this trader has then been reduced by one. We assume that inflowing traders are heterogeneous with respect to deadlines.

A trader with deadline i has in total i opportunities to come to an agreement with his assigned partner. After i disagreements the trader receives zero surplus and disappears from the market. In contrast to our companion paper Hurkens and Vulkan (2006), we assume here that the traders in each pair have private information about their deadline. Hence, the proposer can not make his offer contingent on the responder's deadline. Of course, the offer may depend on the deadline of the proposer and could thereby signal valuable information to the responder. For example, an offer that is only made by proposers with a deadline of 2 would inform the responder that the proposer has in fact a deadline that is almost expiring. It could be optimal for the responder to reject and make a counter offer next period, demanding (almost) everything for herself. Allowing for alternating offers within the same pair with two-sided incomplete information would complicate the analysis and possibly yield a plethora of equilibria. We avoid this by insisting that a pair is broken up exogenously after the first rejection and that the probability of meeting the same partner is zero.

Although entry is assumed to be exogenous and constant over time, the total mass of buyers and sellers present in the market may change over time because exit is partly endogenous. On top of that, the proportion of traders with short deadlines may change over time if traders with short deadlines are more likely to come to agreements than traders with long deadlines. We will be interested in the stationary state of the model, where the total mass of traders and the relative frequencies of deadlines remains constant over time. This allows us to focus on how the distribution of deadlines affects payoffs, the existence of delays and the possibility of missed deadlines.

We define and show the existence of a stationary equilibrium. We characterize, in terms of the distribution of deadlines of traders who flow into the market in every period, when rejections and delays will occur. We show that in equilibrium deadlines may be missed altogether, even when traders are rematched very frequently. Although traders with deadline 1 will always accept the proposal received, it is possible that such traders make proposals that are sometimes rejected, which leads them to miss their deadline. On the other hand, when no delay occurs in equilibrium, all traders receive the same expected payoff, independent of their deadline. We illustrate the results by analyzing the case of two possible deadlines in detail, for any distribution. This special case indicates that deadlines being missed is not uncommon, and that the efficiency loss caused by it can be quite substantial. We then propose and discuss a number of ways of reducing or eliminating this inefficiency through the introduction of other mechanisms or markets which will, in equilibrium, only attract traders with low deadlines. However, as long as there are costs associated to the introduction of these alternative mechanisms, the inefficiency cannot be completely be eliminated but can be drastically reduced.

Although deadlines seem a very natural way of expressing time pressure and even though bargaining partners are often tied to personal deadlines, the theory of bargaining has not had much to say about how deadlines affect bargaining. In contrast to the current (and our companion) paper, this literature considers two person bargaining with a common deadline.[1] Moore (2005) shows experimentally that many subjects do not realize that the imposition of a time constraint in bilateral bargaining affects their bargaining partner in the same way. Most subjects expect the imposition of a strict deadline to worsen their expected bargaining outcome. Similarly, Moore (2004) shows experimentally that many subjects prefer not to reveal their deadline when this is private information, even though revelation turns out to improve bargaining outcomes.

Our paper relates to the large literature on dynamic matching and bargaining, beginning with Rubinstein and Wolinsky (1985) and summarized in Gale (2000). A dynamic matching and bargaining game constitutes a natural model of bargaining when there are many traders on both sides of the market. Most closely related to our papers is Bose (1996), who considers a dynamic matching and bargaining market in which traders either have a relatively high or a relatively low discount factor. He shows that for some parameters delay occurs since patient traders wait until they meet an impatient trader. Apart from our focus on deadlines rather than discount factors, an important difference is that our model allows for a long and a short side of the market, while Bose (1996) requires complete symmetry in order to obtain existence of a stationary equilibrium. Another difference is that the bargaining behavior of a trader in our model changes over time, while in Bose (1996) it remains constant. Finally, the present paper assumes private information while Bose (1996) assumes that there is perfect information about discount factors. Samuelson (1992) considers buyers and sellers with private, heterogenous valuations and shows that two traders who could realize a positive surplus from trading, may decide to break up negotiations and look for alternative partners, with which they can make even more profitable agreements. A distinction between Samuelson (1992) and our paper is that in the latter the surplus to be divided is constant in all pairs and only the disagreement point differs from pair to pair, while in the former both the bargaining set and the disagreement point change. A further difference is that in our model the situation of a particular trader, even in the steady state, changes from period to period as his deadline is approaching. In Samuelson (1992) the situation of any particular trader remains the same over time, as long as the market is in steady state. This implies that any particular trader makes the same offers and uses the same cutoff level to determine to accept or reject proposals.

The rest of the paper is organized in the following way: Section 2 presents the general model. In section 3 we define and show existence of stationary equilibria and analyze their properties. Section 4 discusses ways of reducing delay. In section 5 we show that the model can be extended to allow for cardinal and distributional asymmetries between sellers and buyers. Proofs are collected in the Appendix.

[1] See our companion paper for a more detailed discussion of this literature that moreover assumes that the deadline is known.

2 The Model

We consider a model with a continuum of sellers and buyers (of mass 1 each) flowing into the market every period. All sellers have one unit of a good they produced at zero cost and all buyers have unitary demands for this good, which they all value at one. The only difference between different traders is their *deadline*. The deadline of a trader is an integer number from $\{1, 2, ..., N\}$ that indicates how many periods are remaining for this trader to conclude a deal. If a trader fails to conclude a deal at the last opportunity he misses his deadline and his utility is zero. That is, a trader with deadline 1 will have to make a deal immediately or his opportunity will be lost. Such a trader will be willing to accept any deal that gives him a positive utility. On the other hand, traders with a high deadline will be able and willing to reject certain deals and wait for better opportunities in the future.

We assume that proportion p_i of the sellers (buyers) that flow into the market place every period has deadline i. The procedure for closing trades is as follows: in each period $t \in Z$ each buyer is matched with a seller. One trader in each pair is chosen at random and becomes the proposer (with probability one half). This trader makes a proposal which can be accepted or rejected. In the first case trade takes place and traders disappear from the market. In the second case no trade takes place and both traders go back to the market and become matched next period (with different partners), as long as their deadlines have not expired. Of course, their deadline will then be reduced by one.

We will be interested in the steady state or stationary equilibrium, which will be defined formally below. A stationary equilibrium is an equilibrium where all buyers (sellers) with the same deadline make and accept the same proposals (independent of the time period t) and where the mass of traders in the market place and the distribution of deadlines among the buyers (sellers) (denoted by q) remains constant over time. There are two different scenarios possible. In the first scenario, which we will refer to as the *no delay case*, trade occurs in each matching. In this case the stationary distribution q of deadline types is simply given by p,. In the second case, which we will refer to as the *delay case*, there is no trade taking place in some matches. In this case the stationary distribution q will be different from the inflow distribution p.

We will assume that traders discount late trades by a factor $\delta \leq 1$. It will become clear later on that the role of the discount factor is not as important as in standard bargaining models. The reason is that, as will be shown, traders with longer deadlines will close (weakly) better deals than traders with shorter ones. This gives traders an incentive to make deals early, even if the discount rate is equal to one. However, if we do not allow for discounting of utilities, there is no cost of having delay, as long as deadlines are never missed. As we will see, in some cases deadlines will be missed so that even without discounting delay is costly and inefficient.

We assume throughout this paper that traders that are matched do not know each other's deadline. The proposal made by one of the traders may thus depend only on his own deadline. In a companion paper we analyze the case where traders have perfect information about each other's deadline.

3 Equilibrium Analysis

A pure strategy of a trader flowing into the market in the current period with a deadline i must specify the offer she will make and the offers she will accept in this period, and also the offers she will make and accept in future periods. In principle, the offers made and accepted in future periods could depend on the personal history of received offers. We will be interested in stationary equilibrium states. We will consider only strategies where all traders with the same deadline employ the same strategy. This implies in particular that the strategy a trader with deadline $i+1$ plans to use in the next period is the same as the one used by a current trader with deadline i. For a given stationary distribution of deadlines, q, and for given trader's strategies, one can compute the expected payoffs for a trader with deadline i. We will denote this by v_i. Since a trader with deadline $i+1$ can mimic one with deadline i, we must have $v_i \leq v_{i+1}$.

In a stationary equilibrium state it must be that a trader with deadline i accepts any proposal that gives her $x > \delta v_{i-1}$ and rejects any proposal that gives her $x < \delta v_{i-1}$, where $v_0 = 0$. When the proposal is exactly equal to δv_{i-1}, the responder is indifferent and, in principle, may use a mixed strategy. However, when the responder is indifferent and accepts with probability strictly less than one, the proposer (with deadline j) could deviate and offer slightly more, which would then be accepted for sure. If this is profitable for the proposer, it must be the case, in equilibrium, that the indifferent responder accepts δv_{i-1} with probability 1. On the other hand, if the proposer does not gain from offering slightly more, it must be the case that the proposer is in fact indifferent between the responder accepting or rejecting. This implies that $1 - \delta v_{i-1} = \delta v_{j-1}$. (Note that this cannot happen when $i = 1$.) But in this case it is profitable for the proposer to make a strictly lower proposal $\delta v_{k-1} < \delta v_{i-1}$. In particular, offering the largest $\delta v_{k-1} < \delta v_{i-1}$ will be a strict improvement. Hence, in an equilibrium configuration, it must be the case that any equilibrium proposal is accepted with probability one by responders that are indifferent.

The proposals that are made in a stationary equilibrium state must be in the set $X = \{\delta v_0, \ldots, \delta v_{N-1}\}$. Namely, offers strictly above δv_{i-1} will be accepted for sure by a trader with deadline i or lower, and therefore it is never optimal for a proposer to make such offers. Since it is possible that $v_i = v_j$ for some $i \neq j$, we will write also $X = \{x_1, \ldots, x_n\}$ with the understanding that $x_k < x_{k+1}$. It will turn out that we will have to allow for proposers randomizing between different proposals. Let $\Delta(X)$ denote the set of probability distributions on a finite set X.

We are now ready to define a stationary subgame perfect equilibrium configuration for the case where deadlines are private information.

Definition 1 *We call* $(z, v, s) = ((z_1, ..., z_N), (v_1, ..., v_N), s) \in \Re_+^N \times \Re_+^N \times \Delta(X)^N$ *a stationary subgame perfect equilibrium configuration (with* T*-delay) if the following holds:*

1. $\sum_{i=1}^N z_i = 1 + T$.

2. $z_N = p_N$ *and for all* $i < N$

$$z_i \;=\; p_i + \frac{1}{2} z_{i+1} \left[\sum_k \left[s_{i+1}(x_k) \sum_{j:\delta v_{j-1} > x_k} q_j \right] \right] + \frac{1}{2} z_{i+1} \left[\sum_j \left[q_j \sum_{k:\delta v_i > x_k} s_j(x_k) \right] \right]$$

   where $q_i = z_i/(1 + T)$.

3. *For all* i

$$\begin{aligned} v_i \;=\; & \frac{1}{2} \sum_k \left[s_i(x_k) \left((1 - x_k) \sum_{j:\delta v_{j-1} \leq x_k} q_j + \delta v_{i-1} \sum_{j:\delta v_{j-1} > x_k} q_j \right) \right] \\ & + \frac{1}{2} \sum_j q_j \left[\sum_k \max\{\delta v_{i-1}, x_k\} s_j(x_k) \right]. \end{aligned}$$

4. $s_i(x_k) > 0$ *implies* $x_k \in \arg\max_{x_l} \{(1 - x_l) \sum_{j:\delta v_{j-1} \leq x_k} q_j + \delta v_{i-1} \sum_{j:\delta v_{j-1} > x_k} q_j\}$.

This definition needs some further discussion. First, z_i denotes the mass of traders with deadline i in the stationary state. The proportion of traders with deadline i is denoted by q_i. If $T > 0$ there is delay in equilibrium. Condition 2 just states that in a stationary state, z_i must be equal to the sum of the mass of new traders with deadline i (p_i), the mass of traders with deadline $i + 1$ that make proposals that will be rejected, and the mass of traders with deadline $i + 1$ that reject proposals. Further, $s_i(x_k)$ denotes the probability that trader i as a proposer will offer x_k to the counterpart. Condition 4 states that only offers that maximize a proposer's expected payoff can be chosen with positive probability. Finally, condition 3 states that the expected payoff of a trader with deadline i equals the average of the expected payoff from being a proposer (which in turn is the sum of expected payoffs of accepted and rejected proposals) and the expected payoff of being a responder, given the strategies s and the probabilities q_j of being matched with a trader with deadline j.

Theorem 2 *For any inflow distribution* p *and any discount factor* $\delta \in (0, 1]$ *there exists a stationary subgame perfect equilibrium configuration.*

Lemma 3 *For $N > 1$ there exists an equilibrium with $v_i = v$ for all i if and only if $p_1 \leq 2(1-\delta)/(2-\delta)$. In this case $v = 1/2$ and no delay occurs.*

The intuition for this result is that when the probability of being matched with a trader with deadline 1 is sufficiently small, it is not optimal to make a very greedy offer of zero, as it will be accepted with a very low probability. When the zero offer is never made, a trader with deadline 1 can mimic the strategy of a trader with deadline 2 and obtain the same payoff. In fact, any trader with deadline $i-1$ can mimic the strategy of a trader with deadline i and obtain the same payoff. The only way in which all traders obtain the same payoff v is when $v = \frac{1}{2}(1-\delta v) + \frac{1}{2}\delta v = 1/2$.

Lemma 4 *Suppose $1 > p_1 > 2(1-\delta)/(2-\delta)$. In any equilibrium there will be delay. The payoffs are strictly increasing in deadlines. Deadlines may be missed. Whether they are missed with positive probability depends crucially on the inflow distribution.*

The intuition for this result is as follows. When the probability of being matched with a trader with deadline 1 is sufficiently high, it is optimal for the trader with the highest deadline to make the greedy zero offer, which will only be accepted by traders whose deadline is about to expire. Thus, in this case delay occurs whenever a trader with the highest deadline is chosen as the proposer and is being matched with a trader with a deadline that is not about to expire. This in turn implies that a trader with deadline $i+1$ will obtain a strictly higher expected payoff than a trader with deadline i, as the first can mimic the latter for i periods, but refuse the greedy offer in the i-th period. He will have then have one extra opportunity to obtain a strictly positive payoff. Of course, the condition in the Lemma is equivalent to $\delta > 2(1-p_1)/(2-p_1)$. Hence, for any distribution with full support there will be delay if traders have a discount factor close to 1.

Whether deadlines are missed or not will depend on the fine details of the inflow distribution. Deadlines are missed only when the proposer has deadline 1 and finds it optimal to make a proposal that the traders with the longest deadlines will reject. This will only occur when there are not too many of such traders with the longest deadline. Clearly, the fraction of traders in the market that will miss their deadline in the next period is thus bounded above by 12.5 per cent. Namely, the probability that a random trader in the market will miss his deadline in the next instant is bounded above by $q_1(1-q_1)/2 \leq 0.125$. However, the probability that a trader will eventually miss his deadline may even be higher than that, since traders with high deadlines may first delay trade and later miss their deadline. The special case of $N = 2$ that will be analyzed in detail in subsection 3.2, shows that as much as 14 per cent of new traders missing their deadline eventually is possible.

Next we show that in any equilibrium the proposals made are weakly decreasing in the deadline of the proposer. That is, the more patient a trader is, the greedier his offers will be. Or phrased alternatively, the offers made by one particular trader become more generous over time. Lemma 3 shows that the result cannot be extended to *strictly* decreasing offers.

Lemma 5 *In any equilibrium (with delay) the equilibrium offers are weakly monotonically decreasing in deadline: If type j weakly prefers offering δv_{i-k} rather than δv_i (for $k > 0$), then type $j+1$ strictly prefers offering δv_{i-k} rather than δv_i.*

3.1 Frequent matches

Delay occurs whenever $1 > p_1 > 2(1-\delta)/(2-\delta)$. Holding the discount factor fixed and assuming a uniform distribution over deadlines, this condition tells us that there will be delay when

$$1 < N < \frac{2-\delta}{2(1-\delta)}.$$

Hence, delay will not occur if there are sufficiently many opportunities to match with an alternative partner.

In many practical situations the true deadline of a person will be given by a calendar date. The number of attempts this person will have to conclude a trade will depend on the frictions in the market. In particular, the amount of time that is consumed between consecutive attempts is what really matters. If matches are formed only once a day, a trader whose deadline expires in a month will have about 30 attempts. If matches are formed every hour, he will have available about $30 \times 24 = 720$ attempts. Will delay disappear when frictions go away, as matches are formed more regularly? Trading through internet makes frequent rematching possible and eliminates frictions.

In order to answer this question, let us assume that deadlines in real time are distributed with cumulative density function F and density function f over the interval $[0,1]$ and that real time between two matches equals $\Delta = 1/N$. Let $r > 0$ be the discount rate so that $\delta = \exp(-r\Delta)$ is the discount factor between consecutive periods. The probability of having deadline 1 equals $p_1 = F(1/N)$. Hence, there is delay if

$$F(1/N) > \frac{2(1-e^{-r/N})}{2-e^{-r/N}},$$

or equivalently, if

$$2 - e^{-r/N} > \frac{2(1-e^{-r/N})}{F(1/N)}.$$

In the limit for $N \to \infty$ this is certainly satisfied if $F(0) > 0$. Let us assume that $F(0) = 0$ and that $f(0) > 0$.

In this case, the condition for delay in the limit reads

$$1 > \lim_{N\to\infty} \frac{-2(r/N^2)\exp(-r/N)}{-1/N^2 f(1/N)} = 2r/f(0).$$

Hence, in the limit as matches are established faster and faster, delay disappears only if $f(0) < 2r$.

3.2 Special case: $N = 2$.

In this subsection we will analyze the special case with just two possible deadlines. It will illustrate the possibility of multiple equilibria and the need to allow for mixed strategies in order to guarantee existence of equilibria. Furthermore, we will be able to give some insight how the inflow distribution affects the *probabilities* of delay and missed deadlines, whereas Lemma 4 only indicates the possibilities of delay and missed deadlines.

Let $N = 2$ and $\delta > 0$ be given. We want to calculate all stationary subgame perfect equilibria for all $p_1 = p \in [0, 1]$. In this case, only offers of 0 and δv_1 will be made in an equilibrium. We will refer to the former as the greedy proposal and to the latter as the generous proposal.

We check first for all pure equilibria. There are three possibilities: (1) both types make the generous offer; (2) both types make the greedy offer; and (3) the short deadline type offers δv_1 and the long deadline type offers 0. From Lemma 5 we know that if it is optimal for the short deadline type to offer 0, then this is also optimal (even uniquely so) for the long deadline type. Hence, it cannot occur that the short deadline type offers 0 while the long deadline type offers δv_1. It turns out that for a range of values of p, no pure SSPE exists. Considering mixed equilibria, there are only two possibilities:(4) a proposer with deadline 1 mixes between the greedy and the generous proposal while a proposer with deadline 2 makes the greedy proposal for sure; and (5) a proposer with deadline 1 makes the generous proposal while the proposer with deadline 2 mixes between the greedy and the generous one. Lemma 5 implies that no other mixed SSPE exist. We will see that in fact case (5) cannot occur.

Lemma 6 *For the case $N = 2$ and $\delta > 0$ we have*

1. *Both types of traders making the generous proposal constitutes an SSPE if and only if*

$$p \in \left[0, \frac{2(1-\delta)}{2-\delta}\right].$$

2. *Both types making the greedy zero offer constitutes an equilibrium if and only if*

$$p \in \left[\frac{4+\delta-\delta^2}{4+3\delta}, 1\right].$$

3. *The traders with deadline 2 making the greedy offer and the traders with deadline 1 making the generous offer constitutes an SSPE if and only if*

$$p \in \left[\frac{2(1-\delta)}{2-\delta}, \frac{2-\delta}{2}\right].$$

4. *Traders with deadline 2 making the greedy offer and traders with deadline 1 using a strictly mixed strategy constitutes an equilibrium if and only if*

$$p \in \left(\frac{2-\delta}{2}, \frac{4+\delta-\delta^2}{4+3\delta}\right).$$

5. *There is no SSPE in which traders with deadline 2 use a strictly mixed strategy.*

We conclude that, in the case of $N = 2$, for all $p \neq 2(1-\delta)/(2-\delta)$ a unique SSPE exists. For an open set of parameter values this SSPE involves proposers with deadline 1 randomizing between the two possible proposals. For $p = 2(1-\delta)/(2-\delta)$ there are exactly two SSPE, and both are in pure strategies. In both, traders with deadline 1 make the generous proposal. Traders with deadline 2 make the generous proposal in one of the SSPE, while they make the greedy proposal in the other. Even though in both SSPE, proposers with deadline 2 are indifferent between the two possible proposals, there are no mixed equilibria in which these traders randomize between the two proposals. Also note that the payoff for traders with deadline 1 (v_1) in these two SSPE differs, and therefore, also the generous proposal will be different. In particular, for $\delta = 0.99$ and $p = 2(1-\delta)/(2-\delta)$, the payoffs are

$$v_1^{\text{gen}} = 0.5 \text{ and } v_1^{\text{greed}} \approx 0.3837.$$

Figure 1 depicts several aspects of the SSPE in the case $N = 2$ and $\delta = 0.99$ for a wide range of p. The small range where there is no delay is not depicted. The green graph indicates the probability that a trader entering the market will miss his deadline (eventually). The red graph depicts T, the mass of delayed traders in the market. This is highest when p is low and proposers with deadline 2 offer zero. The light blue graph is the expected payoff a trader with deadline 1, i.e. v_1. Recall that the generous proposal equals δv_1. For large values of p the generous proposal is quite high, and, is in equilibrium not offered. The most interesting

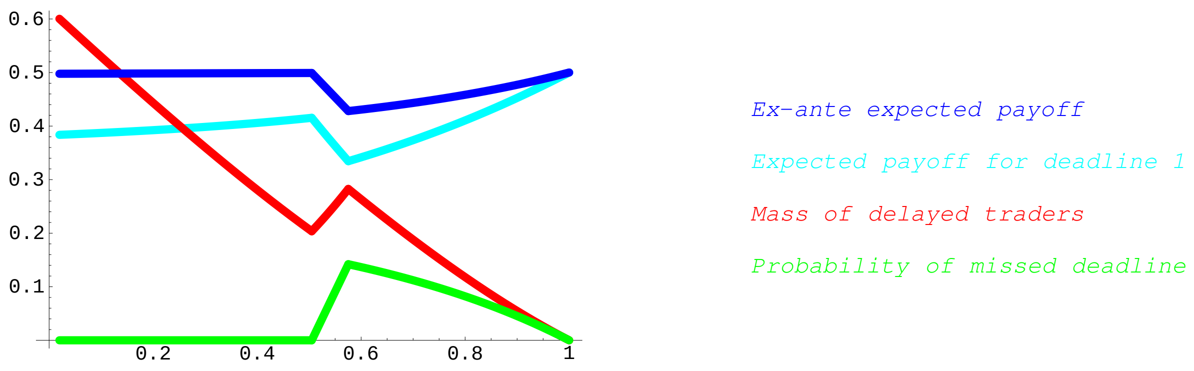

Figure 1: Results example $N = 2$.

graph is the dark blue one. This indicates the ex-ante expected payoff of a trader entering the market. Since in the case of no delay this is equal to one half, values strictly below 0.5 indicate inefficiency. Efficiency loss is maximal for the smallest value of p such that both traders make the greedy proposal with probability 1 in equilibrium. Realized expected payoff is approximately equal to 86 per cent of potential gain. Note that inefficiency is much higher when deadlines are missed then when there is a lot of delay.

4 Reducing delay and improving efficiency

We have seen that inefficient delay will occur for many distributions of deadlines. The examples have also demonstrated that the amount and cost of inefficiencies can be substantial, as deadlines may be missed altogether. In this section we ask the question if there exist mechanisms that would yield efficient outcomes. In particular, is there a role for commercial intermediaries to offer alternative trading opportunities yielding efficient outcomes?

The central idea here will be to protect the traders with the shortest deadlines from the more patient traders who try to exploit them. If an intermediary would have a costless technology to distinguish the former traders from the latter, things would be rather straightforward. Namely, by allowing only traders with expiring deadlines to participate and matching them randomly, all participants would come to agreement immediately and receive an expected payoff of one half. Moreover, traders with deadline 1 would voluntary participate in this organized *mini-market*. In fact, as long as delay takes place in the decentralized random matching market, these traders would be willing to pay a fee $f > 0$ for participation, so that the intermediary can recover possible transaction costs. If all traders with deadline 1 do this, the other traders would be randomly matched among themselves, and since none of them has deadline 1, there would be no delay and offers are always equal to $\delta/2$ (See lemma 3.) However, if participation in the mini-market requires paying a positive participation fee $f > 0$, traders with deadline 1 would prefer to deviate and return to the decentralized random matching market. Hence, if a positive fee has to be paid (for example, because the technology to distinguish traders with deadline 1 from the rest is costly), only a fraction of traders with deadline 1 should participate in the mini-market to have a stable outcome. This fraction would be determined by an indifference condition (between the two markets) for traders with deadline 1. In principle, the intermediary would be able to set a profit maximizing fee, given the demand function for the protection offered. Note that some delay must remain in the decentralized market.

The assumption that an intermediary can distinguish traders with deadline 1 is presumably too strong. Traders could perhaps be induced to reveal their deadline truthfully. For example, participants may be required to deposit a considerable amount of money M that will only be reimbursed if they come to an immediate agreement with the assigned partner. This would deter traders with high deadlines to enter the mini-market. However, this may create a hold-up problem between traders with deadline 1: a proposer may ask for more than the full pie since a rejection would yield the responder a payoff of $-M$.

An alternative and more practical way to reduce delay and increase efficiency is for the intermediary to offer to buy at price $1/2 - f$ and to sell at price $1/2 + f$. For small $f > 0$, traders with deadline 1 would prefer this posted price mechanism if the decentralized market involves delay and exploitation. As before, it is not possible in equilibrium that *all* traders with deadline 1 trade at the posted prices, since then the decentralized matching market would yield an expected payoff of $1/2$. Hence, the posted prices will attract just enough traders with deadline 1 so that these traders are exactly indifferent between the posted price and the decentralized market. Since some delay then still occurs in the decentralized matching market, expected payoffs are strictly increasing in deadlines (see lemma 4) so that traders with deadlines higher than 1 will strictly prefer the decentralized matching market over the posted price.

As long as a positive fee has to be paid by traders with deadline 1, delay can be reduced but not completely eliminated. However, if the fee is not too high, one can at least guarantee that no trader will miss his deadline. Namely, if traders with deadline 1 have a positive probability of missing their deadline, they will for sure miss their deadline in the event that they are chosen as a proposer and being matched with a trader with the highest deadline. Consequently, their expected payoff is strictly bounded away from $1/2$. For low enough fees no trader will thus miss his deadline.

5 Asymmetric markets

We have assumed throughout the paper that the number of sellers and buyers flowing into the market is the same. We also assumed that the distribution of deadlines is the same for buyers and sellers. This allowed us to simplify the exposition. However, this simplification is not crucial to our main results and both of these assumptions can be relaxed. In this section we discuss briefly how this can be done and why our main results will remain to hold.

Let p_i denote the mass of sellers with deadline i flowing into the market, and let $(1+b)p'_i$ denote the mass of buyers with deadline i flowing into the market. Without loss of generality we may assume that $\sum_{i=1}^{N} p_i = 1$ and $\sum_{i=1}^{N'} p'_i = 1$. If $b > 0$ then the buyers form the long side of the market. Obviously, in this case there will always be buyers who will not get matched in any single round. That means that there will certainly be delay among the buyers, and also that some buyers may miss their deadline, since some buyers may never be matched with a seller before his deadline expires. However, we will be interested in the possibility of delay and missed deadlines on the (short) side of the sellers.

The definition of a stationary subgame perfect equilibrium configuration can be generalized in a straightforward manner. Namely, it will be a tuple $(z^B, z^S, w^B, w^S, M^B, M^S)$, where z_i^B and z_j^S denote the mass of buyers (sellers) with deadline i (j) and where w_i^B and w_j^S denote the expected payoff of a buyer (seller) with deadline i (j). The $N' \times N$ matrix M^B indicates the probability of proposal $(1 - \delta w_{j-1}^S, \delta w_{j-1}^S)$ being made by a buyer with deadline i. Similarly, the matrix M^S indicates the probabilities of proposals by the sellers. If $\sum z_j^S = 1 + T$ and $T > 0$, there will be delay among sellers. The stationarity condition for buyers must now take into account not only that some buyers remain from the previous period because they made or rejected unacceptable proposals, but also those buyers who were not matched in the last period. Since the total mass of sellers and buyers is bounded above by N and $(1 + b)N'$, respectively, the model does not explode and existence of a stationary subgame perfect equilibrium can be shown by means of Kakutani's fixed point theorem.

On the long side of the market the payoffs are strictly increasing in deadlines. This follows from the fact that there is a positive probability of not being matched, and therefore, of missing one's deadline. A buyer with deadline $j + 1$ can guarantee a payoff strictly higher than that of a buyer with deadline j by mimicking the latter's strategy, since he will have one last and additional opportunity to obtain a strictly positive payoff. On the short side of the market the payoffs are strictly increasing in deadlines if and only if delay occurs on that side. Namely, if no delay occurs, all sellers will make and accept the same offers. Therefore, all sellers will obtain the same payoff in this case. If delay occurs in equilibrium ($T > 0$), payoffs will be strictly increasing for the same reasons as in the symmetric case. The payoffs will typically be lower on the long side of the market because of the positive probability of not being matched. When the ratio of buyers per seller is very high ($b >> 0$), buyer's payoffs will be close to zero. In this extreme case there is no reason for sellers to delay since they can appropriate almost all of the surplus, even when matched with a buyer with a long deadline.

6 Conclusions

The Internet is playing an increasingly important role as a platform where traders interact. More and more sites, including the market leader eBay, now allow buyers and sellers to directly negotiate prices between them. The two key features of these interactions are (a) a continuous and exogenous arrival of new traders, and (b) traders remain anonymous. Given the anonymity it is natural to assume that traders do not know each other's time preference. We model time preference by means of heterogeneous deadlines, which is very natural in this context, as many online interactions are conducted via software agents. (See for example Sandholm and Vulkan (2000) for a discussion of the role of deadlines in e-commerce applications using software agents.) This paper studies a stylized model based on these assumptions.

Because people with short deadlines will accept very low offers, traders with longer deadlines have incentives to wait to be matched with the impatient traders, and, in equilibrium, will do so for certain parameters. In our companion paper, Hurkens and Vulkan (2006), we have shown this happens if deadlines are commonly known (that is, each trader can observe the deadline of the person she is matched with). In this paper we show that a similar result holds for the more realistic case of deadlines that are private information.

So heterogeneous deadlines cause delay. This has been shown before in experimental setups (Roth *et al.*, 1988), and in studies which look at behavior in online auctions (Roth and Ockenfels, 2002). Even worse, when deadlines are private information, deadlines may be missed which causes a considerable efficiency loss.

In this paper we showed the existence of a steady state equilibrium and provided partial comparative statics on delay and inefficiency. Unlike the perfect information case considered in Hurkens and Vulkan (2006), traders in this model cannot condition their offers on the deadlines of the person they are matched with. Still, if the probability of being matched with someone whose deadline is about to expire is not too small, then traders will make offers that are rejected by those who have high deadlines. In this case we show that traders with longer deadlines fare strictly better. If the probability of being matched with a trader with the lowest deadline is sufficiently small this will no longer be the case and, in equilibrium, all traders will receive the same expected payoffs.

This last result provides a hint of how the problem of delay and missed deadlines can be approached: If there is a way of separating the low types from the others, then the incentive to delay will disappear. In section 4 we discussed a number of ways this can be achieved. However, we also showed that the inefficiency cannot be fully eliminated whenever the technology to distinguish between traders with different deadlines is not costless.

Direct trading between buyers and sellers on the Internet is likely to increase and become a regular feature of everyday economics. An understanding of the underlying incentive structure in such interactions is therefore important. This model, while highly stylized, is a step in this direction.

Appendix: Proofs

Proof of Theorem 2

Let $Z = \{z \in \Re_+^N : \sum_i z_i \geq 1 \text{ and } z_i \leq N + 1 - i\}$ and let $W = \{w \in \Re_+^N : w_1 \leq w_2 \leq \ldots \leq w_N \leq 1\}$. Let $\mathcal{M}$ denote the set of all $N \times N$ matrices with entries in the interval $[0, 1]$ whose row sums are equal to 1. For any $w \in W$, let $X(w) = \{\delta w_0, \delta w_1, \ldots, \delta w_{N-1}\}$ denote the set of (potentially optimal) offers. (Here $w_0 = 0$.) The entry M_{ij} of the matrix M is to be interpreted as the probability with which a proposer with deadline i offers δw_{j-1}.

Consider the following correspondence $G : Z \times W \rightrightarrows Z \times W \times \mathcal{M}$:

$$G(z, w) \quad = \quad \{(z, w, M) : M_{ij} = 0 \text{ if } \delta w_{j-1} \notin \arg\max_{x \in X(w)} \{(1 - x) \sum_{j:\delta w_{j-1} \leq x} z_j + \delta w_{i-1} \sum_{j:\delta w_{j-1} > x} z_j\}.$$

The correspondence G adds (mixed) strategies of proposers that are myopically optimal. That is, they are optimal given the distribution of deadlines implied by z, and under the further assumptions that a responder with deadline j accepts proposal x if and only if $x \geq \delta w_{j-1}$ and that rejected proposals yield proposer with deadline i an expected payoff equal to w_{i-1} one period later.

We furthermore define the following mapping $H : Z \times W \times \mathcal{M} \to Z \times W$:

$$H(z, w, M) = (\tilde{z}, \tilde{w})$$

where $\tilde{z}_N = p_N$ and for $i < N$,

$$\tilde{z}_i = p_i + \frac{z_{i+1}}{2 \sum_j z_j} \left(\left[\sum_k M_{i+1\ k} \sum_{j:\delta w_{j-1} > \delta w_{k-1}} z_j \right] + \left[\sum_j \left[z_j \sum_{k:\delta v_i > \delta w_{k-1}} M_{jk} \right] \right] \right),$$

and, for all i,

$$\begin{aligned}\tilde{w}_i \quad = \quad & \frac{1}{2 \sum_j z_j} \left(\sum_k \left[M_{ik} \left((1 - \delta w_{k-1}) \sum_{j:\delta v_{j-1} \leq \delta w_{k-1}} z_j + \delta w_{i-1} \sum_{j:\delta v_{j-1} > \delta w_{k-1}} z_j \right) \right] \right) \\ & + \frac{1}{2 \sum_j z_j} \left(\sum_j z_j \left[\sum_k \max\{\delta v_{i-1}, \delta w_{k-1}\} M_{jk} \right] \right).\end{aligned}$$

The mapping H recalculates the mass of traders with different deadlines from the proposal strategies given by M (and given the responders' strategies described before) and updates the expected payoff of traders. Note that $\tilde{z}_i \leq p_i + z_{i+1} \leq 1 + N + 1 - (i+1) = N + 1 - i$ and that $\tilde{w}_i \leq \tilde{w}_{i+1} \leq 1$ when $w_{i-1} \leq w_i$ so that H really maps into $Z \times W$.

We now combine G and H to construct a correspondence $F : Z \times W \rightrightarrows Z \times W$ as follows:

$$F(z, w) = \{H(z, w, M) : (z, w, M) \in G(z, w)\}.$$

F is an upper semi-continuous correspondence from a non-empty, compact, convex set $Z \times W$ into itself such that for all $(z, w) \in Z \times W$, the set $F(z, w)$ is convex and non-empty. Convexity of $F(z, w)$ is of course immediate in the case of a singleton set. Suppose $(\tilde{z}, \tilde{w}) = H(z, w, M)$ and $(\tilde{z}', \tilde{w}') = H(z, w, M')$ are two different elements of $F(z, w)$ and let $\alpha \in [0, 1]$. By the definition it follows immediately that $(z, w, \alpha M + (1 - \alpha)M') \in G(z, w)$. Because of the linearity in M, it is straightforward that

$$\alpha(\tilde{z}, \tilde{w}) + (1 - \alpha)(\tilde{z}', \tilde{w}') = H(z, w, \alpha M + (1 - \alpha)M').$$

Hence, $\alpha(\tilde{z}, \tilde{w}) + (1 - \alpha)(\tilde{z}', \tilde{w}') \in F(z, w)$ and applying Kakutani's fixed point theorem delivers the required result. □

Proof of Lemma 3

First, assume the inequality is satisfied and consider the following strategies. All types offer as a proposer $\delta/2$ to the opponent and keep $1 - \delta/2$ for themselves. Types with deadlines $i > 1$ accept any proposal that yields them at least $\delta/2$. Type 1 accepts any proposal that yields him a nonnegative payoff. It is clear that these strategies yield all traders an expected payoff of 1/2. Also, given the proposer's strategies, it is optimal to accept any proposal equal to or above $\delta/2$ and to reject (in the case of deadlines higher than 1) any lower proposals. For a trader with deadline 1 it is obviously optimal to accept any nonnegative offer. The only remaining question is whether some trader could do better by making a different proposal. Given the responder's strategies, the only alternative strategy that could possibly give a higher payoff would be to offer 0 (in the hope of being matched with a trader whose deadline is about to expire). Conditional on being a proposer (with deadline $i > 1$) following the outlined strategy yields $1 - \delta/2$. Offering 0 will only be accepted by traders with deadline equal to 1, so this yields an expected payoff of $p_1 \times 1 + (1 - p_1) \times \delta v_{i-1} = \delta/2 + p_1(1 - \delta/2)$. The equilibrium condition is thus

$$1 - \delta/2 \geq \delta/2 + p_1(1 - \delta/2)$$

or, equivalently,

$$p_1 \leq 2(1-\delta)/(2-\delta).$$

Second, suppose the inequality is not satisfied and suppose there exists an equilibrium with $v_i = v$ for all i. Clearly, $v \leq 1/2$. In such equilibrium offers above δv must be accepted and offers below must be rejected (except for traders with deadline 1 who will accept any nonnegative offer). Therefore, the only offers that can possibly be made in equilibrium are δv and 0. If 0 is never offered then it follows that $v = 1/2$ but then traders with high deadline are better off offering 0, as we have seen before. Hence, some traders must propose 0 with positive probability. But that implies that $v_i < v_{i+1}$ for all i as a trader with deadline $i+1$ can imitate a trader with deadline i but has an extra chance to get a positive payoff in the event of being proposed i times an offer of zero. □

Proof of Lemma 4

Traders with deadline $j+1$ can always use the same strategy that traders with deadline j use. Hence, $v_{j+1} \geq v_j$. From the previous lemma we know that not all traders receive the same payoff. Hence, for some deadline k we have $v_1 \leq \ldots \leq v_k < v_{k+1} \leq \ldots \leq v_N$. This implies that a trader with deadline k cannot imitate the trader with deadline $k+1$. Hence, there must be a positive probability that a trader with deadline $k+1$ will still be in the market when his deadline has reduced to 1. This in turn implies that for any trader with deadline $j < k+1$ there is a positive probability that he will remain in the market for j periods. In particular, this is the case for a trader with deadline 2. Clearly, this means that delay occurs with positive probability.

First we show that $v_1 < v_2$. This is obvious if the zero offer is made with positive probability, since then a trader could just mimic the behavior of a trader with deadline 1, except for the case where a zero offer is made, which should be rejected. Similarly, if the trader with deadline 1 makes an offer in equilibrium which is rejected with positive probability, then the trader with deadline 2 can mimic a trader with deadline 1. In case of being chosen as a proposer and the proposal being rejected, the trader with deadline 2 will have another chance to obtain a positive payoff. Hence, also in this case we must have $v_1 < v_2$. So let us assume that the zero offer is not made and that traders with deadline 1 make a proposal that is accepted for sure. Then the lowest offer that can be made in equilibrium equals δv_1, which a trader with deadline 2 will accept with probability 1 in equilibrium. We know that a trader with deadline 2 will sometimes delay in equilibrium. This can only happen when he makes a proposal that is rejected by some trader(s), let us say, $\delta v_{m-1} < \delta v_{N-1}$. We can only have that $v_1 = v_2$ if in fact trader 2, as a proposer, is indifferent between making the offers δv_{m-1} and δv_{N-1}. But if this is the case, trader k can mimic the behavior of a trader with deadline $k+1$ as long as the remaining deadline is strictly above 1, and make the proposal that is certainly accepted when the deadline has reduced to 1. In this way trader k can obtain the same payoff as trader $k+1$, which is a contradiction. Hence, we have established that $v_1 < v_2$.

It follows immediately that for all traders with $j < k$ that $v_j < v_{j+1}$. Namely, trader $j+1$ can mimic trader j as long as the remaining deadline is above 2, and use the equilibrium strategy of a trader with deadline 2 when the deadline has reduced to 2. In this way trader $j+1$ guarantees a payoff strictly higher than what trader j can get.

We thus have either that (i) $v_1 < v_2 < ... < v_N$ or (ii) there exists a deadline $j > k$ such that $v_1 < ... < v_j = v_{j+1} \leq ... \leq v_N$.

Suppose we are in case (ii). As argued before, it must be the case that there is a positive probability of delay for all traders with deadline i such that $1 < i < j+1$. Since the trader with deadline $j+1$ could imitate the trader with deadline j, but cannot do better than him, it means that the zero offer is never made. (Namely, if the zero offer is made with positive probability, then he could do strictly better by not accepting the zero offer when his deadline has reduced to 2.) Consider now the possibility of trader $j+1$ imitating trader j for one period. If a deal is concluded, he would obtain the same payoff as the trader with deadline j in that circumstance. On the other hand, if there is delay (which happens with positive probability, then his future expected payoff is $v_j > v_{j-1}$. Hence, trader $j+1$ can guarantee a strictly higher payoff than j and case (ii) cannot occur. We conclude that in any equilibrium $v_1 < ... < v_N$.

Proof of Lemma 5. We already know that in an equilibrium without delay all traders make the same offer of $\delta/2$. Consider an SSPE with delay such that $v_{j-1} < v_j$ for all j. Denote $Q_m = q_1 + ... + q_m$ for any $1 \leq m \leq N$. Then $Q_m < Q_{m+1}$ and $Q_N = 1$.

$$\begin{aligned} (1-\delta v_i)Q_{i+1} + \delta v_{j-1}(1-Q_{i+1}) &\leq (1-\delta v_{i-k})Q_{i-k+1} + \delta v_{j-1}(1-Q_{i-k+1}) \\ \Leftrightarrow Q_{i+1}(1-\delta v_i - \delta v_{j-1}) &\leq Q_{i-k+1}(1-\delta v_{i-k} - \delta v_{j-1}) \\ \Rightarrow Q_{i+1}(1-\delta v_i - \delta v_j) &< Q_{i-k+1}(1-\delta v_{i-k} - \delta v_j) \\ \Leftrightarrow (1-\delta v_i)Q_{i+1} + \delta v_j(1-Q_{i-k+1}) &< (1-\delta v_{i-k})Q_{i-k+1} + \delta v_j(1-Q_{i-k+1}) \end{aligned}$$

The first inequality states that a trader with deadline j weakly prefers to offer δv_{i-k} rather than δv_i. The last inequality says that a trader with deadline $j+1$ strictly prefers to offer δv_{i-k} rather than δv_i.

Proof of Lemma 6.

(1). Offers will always be accepted so there is no delay and $v_1 = v_2 = 1/2$. The stationary distribution is given by the inflow distribution: $(z_1, z_2) = (p, 1-p)$. The only plausible deviation is where deadline 2 offers 0. This would yield him (conditional on being proposer) $p + \delta(1-p)/2$. Sticking to the strategy yields

$1-\delta/2$. Hence, these strategies constitute an equilibrium if and only if

$$p+\delta(1-p)/2 \leq 1-\delta/2 \Leftrightarrow p \leq \frac{2(1-\delta)}{2-\delta}.$$

For $\delta = 0.99$ this means (approximately) $p \in [0, 0.0189]$.

(2). Suppose both types offer zero when it is their turn to make a proposal. Such proposals are only accepted by the short deadline types. Hence, there is delay whenever the responder has a long deadline. We will express the payoffs and steady state masses as functions of p and δ. Afterwards we impose the optimality (and feasibility) conditions. First, we have $z_2 = 1-p$ and $z_1 = 1+T-z_2 = T+p$. Also, the steady state condition reads

$$z_1 = p + z_2\left(\frac{1}{2}+\frac{1}{2}\frac{z_2}{T+1}\right),$$

since all traders with deadline 2 who are responders and those who are proposers but matched with deadline 2 type will remain in the market. From this we solve

$$T = \frac{1}{4}(-1-p+\sqrt{17-22p+9p^2}).$$

Note that $T > 0$ for all $p < 1$. Given the steady state distribution and the strategies, payoffs equal

$$v_1 = \frac{1}{2}\frac{z_1}{1+T}$$

and

$$v_2 = \frac{1}{2}\frac{z_1}{1+T} + \left(1-\frac{1}{2}\frac{z_1}{1+T}\right)\delta v_1 = v_1 + (1-v_1)\delta v_1.$$

This follows since the payoff for a deadline 1 trader equals the probability of becoming proposer times the probability of meeting another deadline 1 trader (times 1). Expressed in terms of p this gives for a trader with deadline 1

$$v_1 = \frac{p+\frac{1}{4}(-1-p+\sqrt{17-22p+9p^2})}{2(1+\frac{1}{4}(-1-p+\sqrt{17-22p+9p^2}))}.$$

It needs to be verified that it indeed is optimal for a trader with deadline 1 to make the greedy proposal rather than the generous proposal of δv_1 (which would be accepted for sure), i.e. that $1-\delta v_1 \leq z_1/(T+1)$. This optimality condition is equivalent to

$$p \geq \frac{4+\delta-\delta^2}{4+3\delta}.$$

For the case $\delta = 0.99$ this means (approximately) $p \geq 0.575]$.

(3). Suppose the short deadline type offers δv_1, while the long deadline type offers 0. Now there is disagreement only in pairs of traders who both have a deadline 2, since a deadline 1 trader always accepts nonnegative proposals and always makes a proposal that will be accepted for sure.We have $z_2 = 1 - p$ and $z_1 = 1 + T - z_2 = T + p$. Also, the steady state condition reads

$$z_1 = p + z_2 \frac{z_2}{T+1}.$$

From this we solve

$$T = \frac{-1 + \sqrt{5 - 8p + 4p^2}}{2}.$$

Note that $T > 0$ whenever $p < 1$. The payoff of a trader with deadline 1 will be equal to

$$v_1 = \frac{1}{2}(1 - \delta v_1) + \frac{1}{2}\delta v_1 \frac{z_1}{1+T},$$

since he obtains $1 - \delta v_1$ as a proposer, while as a responder he only makes a positive payoff (namely, δv_1) when matched with a trader with deadline 1. From this equation one can resolve v_1, so that by substitution of z_1 and T we obtain

$$v_1 = \frac{1 + \sqrt{5 - 8p + 4p^2}}{2(1 + \delta - \delta p + \sqrt{5 - 8p + 4p^2})}.$$

The payoff of a trader with deadline 2 can be found by substitution in

$$v_2 = \frac{1}{2}\frac{z_1}{1+T} + (1 - \frac{1}{2}\frac{z_1}{1+T})\delta v_1.$$

It needs to be verified that it indeed is optimal for the trader with deadline 2 to make the greedy proposal, i.e. that $1 - \delta v_1 \leq z_1/(1+T) + (1 - z_1/(1+T))\delta v_1$. Furthermore, it needs to be verified that it is optimal for the trader with deadline 1 to make a generous proposal, i.e. that $1 - \delta v_1 \geq z_1/(1+T)$. The first optimality condition is equivalent to

$$p \geq \frac{2(1-\delta)}{2-\delta}.$$

The second optimality condition is equivalent to

$$p \leq \frac{2-\delta}{2}.$$

Hence, the strategies constitute a stationary subgame perfect equilibrium if and only if

$$p \in \left[\frac{2(1-\delta)}{2-\delta}, \frac{2-\delta}{2}\right].$$

Note that this interval has non-empty interior whenever $\delta > 0$.

For the particular case of $\delta = 0.99$, such an SSPE exists if and only if $p \in [0.0189, 0.505]$.

Also note that for all $\delta > 0$,

$$\frac{2-\delta}{2} < \frac{4+\delta-\delta^2}{4+3\delta}.$$

Hence, there is an open interval of values for p such that no stationary subgame perfect equilibrium **in pure strategies** exists. We now turn to the matter of all possible SSPE in mixed strategies. Note that if it is weakly optimal for a proposer with deadline 1 to be greedy, that it is strictly optimal for a proposer with deadline 2. Therefore, the only possibilities for SSPE in mixed strategies are: (4) a proposer with deadline 1 mixes between greedy and generous proposal while a proposer with deadline 2 makes the greedy proposal for sure; and (5) a proposer with deadline 1 makes a generous proposal while the proposer with deadline 2 mixes between the greedy and the generous one.

(4). Let $x \in [0, 1]$ denote the probability with which the proposer with deadline 1 makes the generous proposal δv_1. We have, as before, $z_2 = 1 - p$ and $z_1 = T + p$. A proposer with deadline 1 must be indifferent between the two possible proposals. Hence

$$\frac{z_1}{1+T} = 1 - \delta v_1.$$

Furthermore, the payoff of a trader with deadline 1 must satisfy

$$v_1 = \frac{1}{2}\frac{z_1}{1+T}(1 + x\delta v_1),$$

since as a proposer he is indifferent, so his payoff is then equal to the probability of meeting a trader with deadline 1 (times 1), while, as a responder, he only obtains a positive payoff (δv_1) when he is matched with a trader with deadline 1 who happens to make the generous proposal (with probability x). From these four equations, one can deduce that there is a unique solution that satisfies $v_1 \geq 0$. The solution is given by

$$T + 1 = \frac{(1-p)(2+\delta-\delta x)\sqrt{4\delta^2 x + (2+\delta-\delta x)^2}}{2\delta},$$

and

$$v_1 = \frac{-2-\delta+\delta x + \sqrt{4\delta^2 x + (2+\delta-\delta x)^2}}{2\delta^2 x}.$$

The steady state condition reads

$$z_1 = p + z_2\left[\frac{1}{2}\left(\frac{z_1}{1+T}(1-x) + \frac{z_2}{1+T}\right) + \frac{1}{2}\left(\frac{z_2}{1+T}\right)\right].$$

Namely, responders with deadline 2 remain in the market if they are matched with traders with deadline 2 or with traders with deadline 1 who make the greedy proposal. Proposers with deadline 2 remain in the market when matched with traders with deadline 2. Using these equalities and the feasibility condition $T \geq 0$, there is a unique solution for x:

$$x = \frac{A - B\sqrt{Q}}{C},$$

where

$$\begin{aligned}
A &= 4 - 8\delta + 2\delta^2 - 8p + 10\delta p + 4p^2 - 2\delta p^2; \\
B &= 2(2 - \delta - 2p); \\
C &= 2(\delta - \delta^2 - 2\delta p + \delta^2 p + \delta p^2); \\
Q &= 1 - 2\delta + 2\delta^2 - 2p + 2\delta p - 2\delta^2 p + p^2 + \delta^2 p.
\end{aligned}$$

Since we have used the indifference condition for traders with deadline 1, these traders are playing optimally. Moreover, since it is weakly optimal for traders with deadline 1 to make greedy proposals, it is strictly optimal for the other traders to do so. Hence, no further optimality conditions need to be imposed. However, one needs to make sure that $0 \leq x \leq 1$. It turns out that

$$0 < x < 1 \Leftrightarrow p \in \left(\frac{2-\delta}{2}, \frac{4+\delta-\delta^2}{4+3\delta}\right).$$

For the case $\delta = 0.99$ we thus have such a mixed equilibrium for (approximately) $p \in (0.505, 0.575)$.

(5). Assume that proposers with deadline 1 make the generous proposal, and that proposers with deadline 2 make the generous proposal with probability $x \in (0, 1)$. We have $z_2 = 1 - p$ and $z_1 = T + p$ (with $T \geq 0$). A proposer with deadline 2 must be indifferent:

$$1 - \delta v_1 = \frac{z_1}{1+T} + \frac{z_2}{1+T}\delta v_1.$$

The steady state condition reads

$$z_1 = p + (1-x) z_2 \frac{z_2}{1+T},$$

since a trader with deadline 2 remains in the market only if he is matched with another trader with deadline 2 and the proposer (whichever of the two) makes the greedy proposal (which occurs with probability $1 - x$).

Furthermore, the payoff for traders with deadline 1 must satisfy

$$v_1 = \frac{1}{2}(1-\delta v_1) + \frac{1}{2}\frac{z_1}{1+T}\delta v_1 + \frac{1}{2}\frac{z_2}{1+T}x\delta v_1.$$

The first term is his payoff from being a proposer; the second term is his payoff from being a responder matched with a trader with deadline 1; the last term is his payoff as a responder matched with a trader with deadline 2. Finally, we have

$$v_2 = \frac{1}{2}\frac{z_1}{1+T} + (1-\frac{1}{2}\frac{z_1}{1+T})\delta v_1,$$

since trader with deadline 2 receives 1 if he is the proposer and matched with a trader with deadline 1, and δv_2 otherwise. These equations have no interior solution for x. The only solutions exists when $p = 2(1-\delta)/(2-\delta)$ and are $x = 1$ and $x = 0$, which corresponds exactly to the pure strategy equilibrium in cases (1) and (3), respectively. □

References

Bose, G. (1996). "Bargaining Economies with Patient and Impatient Agents: Equilibria and Intermediation," *Games and Economic Behavior*, vol. 14, pp. 149-172.

Gale, D. (2000). *Strategic Foundations of General Equilibrium: Dynamic Matching and Bargaining Games.* Cambridge and New York. Cambridge University Press.

Hurkens, S. and N. Vulkan (2006). "Dynamic Matching and Bargaining: The Role of Deadlines," Discussion Paper, Oxford and CREA.

Moore, D.E. (2004). "Myopic prediction, self-destructive secrecy, and the unexpected benefits of revealing final deadlines in negotiation," *Organizational Behavior and ¿Human Decision Processes*, Vol. 94, pp. 125-139.

Moore, D.E. (2005). "Myopic biases in strategic social prediction: why deadlines put everyone under more pressure than everyone else," *Personality and Social Psychology Bulletin*, Vol. 31, pp. 668-679.

Roth, A.E., Murnighan, J.K. and Schoumaker, F. (1988). "The Deadline Effect in Bargaining: Some Experimental Evidence," *American Economic Review*, vol. 78, pp. 806-823.

Roth, A.E., and A. Ockenfels (2002). "Late Minute Bidding and the Rules for Ending Second-Price Auctions: Evidence from eBay and Amazon on the Internet." *American Economic Review*, 92(4), pp. 1093-1103.

Rubinstein, A. (1982). "Perfect Equilibrium in a Bargaining Model," *Econometrica*, vol. 50, pp. 97-109.

Rubinstein, A. and A. Wolinsky (1985). "Equilibrium in a market with sequential bargaining.," *Econo-*

metrica, vol. 53, pp. 1133-1150.

Samuelson, L. (1992). “Disagreement in Markets with Matching and Bargaining,” *Review of Economic Studies*, vol. 59, pp. 177-185.

Sandholm, T. and N. Vulkan (2000). “Bargaining with Deadlines”, *proceedings of the National Conference on Artificial Intelligence (AAAI)*, Orlando, FL.