A Retail Benchmarking Approach to Efficient Two-Way Access Pricing: Termination-Based Price Discrimination with Elastic Subscription Demand *

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Abstract

We study how access pricing affects network competition when consumers' subscription demand is elastic and networks compete with non-linear prices and can use termination-based price discrimination. In the case of a fixed per minute termination charge, our model generalizes the results of Gans and King (2001), Dessein (2003) and Calzada and Valletti (2008). We show that a reduction of the termination charge below cost has two opposing effects: it softens competition and it helps to internalize network externalities. The former reduces consumer surplus while the latter increases it. Firms always prefer termination charge below cost, either to soften competition or to internalize the network effect. The regulator will favor termination below cost only when this boosts market penetration.

Next, we consider the retail benchmarking approach (Jeon and Hurkens, 2008) that determines termination charges as a function of retail prices and show that this approach allows the regulator to increase subscription without distorting call volumes. Furthermore, we show that an informed regulator can even implement the first-best outcome by using this approach.

Keywords: Networks, Access Pricing, Interconnection, Regulation, Telecommunications

JEL numbers: D4, K23, L51, L96

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1 Introduction

In most countries in the world, there is more than one telecommunication network. There is thus competition on the retail level for customers. In order to provide their customers with the possibility to connect to subscribers from rival networks, network operators need to cooperate in terms of terminating calls from rival networks. This requires agreement over the termination charges: How much should operator A pay to operator B in case a call originates from network A but terminates on network B? Since this termination charge enters as a cost (for off-net calls), it affects competition in the retail market. Since each network is basically a monopolist in the market for termination of calls directed to its own customers, in the absence of any regulation, termination charges could be set inefficiently high.[1] There is thus a need to either regulate the termination charge or to have firms negotiate and agree on some termination charge. The latter raises the concern whether the termination charges firms would agree upon (namely the ones that maximize profits) are efficient from a social point of view. Armstrong (1998) and Laffont, Rey and Tirole (1998a) show that, when firms compete in linear prices, bilateral agreements on termination charges lead to monopolistic retail prices, even though there is retail competition. In contrast, Laffont, Rey and Tirole (1998a) find that termination charges lose their collusive power when firms compete in non-linear prices but without termination-based price discrimination, because of a waterbed effect: higher usage prices lead to lower fixed fees. In fact, they show that profits are completely independent of termination charge. This profit neutrality result seems to be a knife-edge result, since some changes in the assumptions of the model may result in firms preferring termination charges above or below termination cost.

In this paper, we study how access pricing affects network competition in a Logit model in which consumers' subscription demand is elastic, networks compete with non-linear prices and can apply termination-based price discrimination. In this sense, we extend the models of Gans and King (2001) and Calzada and Valletti (2008) (who consider inelastic subscription demand) and Dessein (2003) (who does not allow for termination-based price discrimination). We study two very different approaches to determine termination charges. First, we consider the standard approach based on a fixed (per-minute) termination charge and study how the termination charge affects profits and social welfare. We find that both the firms

[1] This would certainly be the case for termination charges for international calls between two operators in different countries. (See Carter and Wright, 1994.) The case is less clear for termination charges between two networks who compete for the same customers. In any case, most national regulators are concerned about too high termination charges and use the argument of significant market power in the market for termination as a justification for intervention.

and the regulator want to depart from cost-based termination charge (and hence want to distort call volumes) in order to affect consumer subscription. In particular, a reduction in termination charge creates two opposing effects, softening competition and internalizing network externalities: the former reduces consumer subscription while the latter expands it. Depending on which effect dominates, there can be conflicts or alignments of interests between the firms and the regulator regarding whether they prefer termination charge below or above cost.

Second, we study the retail benchmarking approach that determines termination charges as a function of retail prices. We extend the approach from the setting without termination-based price discrimination and with inelastic subscription demand (considered in Jeon and Hurkens, 2008) to the setting with termination-based price discrimination and with elastic subscription demand. We show that for a given fixed (reciprocal) termination charge, we can find a family of access pricing rules that induce firms to charge on-net price equal to on-net cost and off-net price equal to off-net cost but the equilibrium fixed fee decreases with the strength of the feedback from the retail prices to access payment. The result implies that the regulator can increase consumer subscription without creating any distortion in call volumes. Our access pricing rules intensify retail competition since a firm can reduce its access payment to rival firms by reducing its average retail prices. The regulator can also use the rule to maintain consumer subscription and reduce the distortion in call volume. Such a rule would improve efficiency and firms' profits. Furthermore, we show that when the regulator and the firms have the same information about demand and cost structure, there is a simple modification of our rule that achieves even the first-best outcome that maximizes market penetration.

In the case of the standard approach, our innovation is to identify the interplay between the two opposing effects associated with a change in termination charge. The result that termination charge below cost softens competition is well-known and discovered by Gans and King (2001) in the context of termination-based price discrimination with inelastic subscription demand: when termination charge is lower than termination cost, on-net price is higher than off-net price and therefore consumers prefer to belong to the smallest network all other things being equal, which reduces firms' incentive to steal customers. Therefore, firms prefer termination charge below cost while the regulator prefers termination charge equal to cost. Recently, Calzada and Valletti (2008)[2] and Armstrong and Wright (2008)[3]

[2]They consider a Logit model with inelastic subscription (i.e. full subscription) while Gans and King (2001) consider the Hotelling model with inelastic subscription.

[3]They consider an extension of the Hotelling model that allows for elastic demand. Although they focus on

find the same result. In order to isolate the effect of internalizing network externalities from the competition-softening effect, we first study a benchmark of "two interconnected islands" in which each island is occupied by a monopolist facing an elastic subscription demand. There is no competition between the two monopolists since consumers cannot move from one island to another. In this benchmark, when a monopolist attracts an additional customer, he creates a positive externality to the other monopolist since the latter's consumers can enjoy off-net calls to the additional customer. Since the two monopolists fail to fully internalize these externalities, the total number of subscribers is smaller than the number when both islands are occupied by one identical monopolist. We find that firms prefer termination charge below cost in order to internalize better the network externalities: a lower termination charge, by increasing the degree of interconnection, increases each subscriber's surplus from off-net calls (for a given number of subscribers), which in turn induces each firm to expand their network size.[4]

It is useful to note that competing firms would like to choose termination charge in order to make the outcome as close as possible to the outcome of a monopolist owning both networks. When termination charge is equal to termination cost, there are two possible cases: the consumer surplus under duopoly can be either larger or smaller than the one under monopoly. We call the first the case with a net business-stealing effect and the second the case with a net network externality effect. In the first case, firms want to decrease consumer surplus while, in the second case, they want to increase consumer surplus. A rather surprising result is that in both cases, firms prefer having termination charge below cost. The reason is that consumer surplus is larger under duopoly than under monopoly exactly when the business stealing effect dominates the network externality effect, so that firms prefer to soften competition, which requires a low termination charge. Consumer surplus is lower under duopoly than under monopoly, exactly when the network externality effect dominates the business stealing effect. Thus firms prefer to internalize the network effect better, which again requires termination charge below cost. The regulator will always

the case in which the fixed to mobile (FTM) termination charge and the mobile to mobile (MTM) termination charge are the same due to arbitrage, when they study the case in which the two can be separately chosen, they find that firms prefer the MTM charge below cost (due to competition-softening effect à la Gans-King) while the regulator prefers the charge above cost.

[4]To some extent, this effect is similar to the result of Katz and Shapiro (1985) that an increase in compatibility among competing networks increases the total number of subscribers. However, in their paper, firms are engaged in Cournot competition and the cost of achieving compatibility is a fixed cost and hence does not directly affect the retail competition (i.e. only demand increasing effect of compatibility remains). In our model, firms compete with non-linear tariffs and interconnection is mediated by the access charge that directly affects retail competition through off-net price: in particular, reducing access charge below cost distorts off-net call volume.

prefer larger consumer surplus so that in the first case it prefers to strengthen competition by means of a termination charge above cost, while in the second case it favors a termination charge below cost (in fact, in this case, the socially optimal access charge is lower than the one preferred by the firms).

Our result in the standard approach based on a fixed termination charge is reminiscent of Dessein (2003)'s finding. Dessein considers a setting without termination-based price discrimination and with elastic demand and shows that firms again prefer to have a termination charge below cost while the regulator prefers a termination charge above cost when the business stealing effect dominates. However, since without termination-based price discrimination, termination charges do not affect profits in the extreme case of inelastic demand, he does not clearly disentangle the two opposing effects as we do. In contrast, with termination-based price discrimination, there is only a business stealing effect in the extreme case of competition with inelastic subscription (as in Gans and King (2001)) while there is only a network externality effect in the other extreme case of two interconnected islands (*i.e.*, no competition with elastic subscription).

Most of the literature that addresses issues related to termination charges in two-way access pricing, considers the termination charge as a fixed (per minute) price.[5] When firms compete in non-linear prices, any termination fee different from termination cost results in inefficient prices. In this paper we depart from this literature and are interested in applying a retail benchmarking approach to the issue of termination prices. In Jeon and Hurkens (2008) we introduced this approach successfully in a framework without termination-based price discrimination and with inelastic subscription demand where firms compete in linear or in non-linear prices. We showed that benchmarking the termination charge a network has to pay to its own average retail price provides firms with incentives to lower their average price as it so reduces termination payments. We showed that by choosing the benchmarking rule appropriately, a regulator can induce Ramsey prices, without having to know consumers' demand function. In the current paper we extend our benchmarking rule to the case where subscription demand is elastic and firms can charge different prices for on-net and off-net calls.

We show that for a given fixed (reciprocal) termination charge, we can find a family of access pricing rules parameterized by $\kappa(\leq 1)$ such that all the access pricing rules in

[5]See, for instances, Armstrong (1998), Armstrong and Wright (2008), Calzada and Valletti (2008), Cambini and Valletti (2008), Carter and Wright (1999, 2003), Dessein (2003, 2004), Gans and King (2000, 2001), Hahn (2004), Hermalin and Katz (2001, 2004), Laffont, Marcus, Rey and Tirole (2003), Laffont, Rey and Tirole (1998a,b), Jeon, Laffont and Tirole (2004), Valletti and Cambini (2005) and Wright (2002).

the family induce firms to charge on-net price equal to on-net cost and off-net price equal to off-net cost but the equilibrium fixed fee decreases with κ where $\kappa = 0$ corresponds to the standard approach based on the fixed termination charge. The result implies that the regulator can increase consumer subscription without creating any distortion in call volumes. The regulator may also use the rule to maintain the number of subscriptions (and therefore consumer surplus) and to reduce the distortion in call volume. Such a rule would improve efficiency and increase profits. Furthermore, we show that when the regulator and the firms have the same information about demand and cost structure, there is a simple modification of our rules that achieves the first-best outcome (i.e. firms charge prices just at costs and consumer subscription is maximized).

The rest of the paper is organized as follows. The next section introduces the logit model formulation of network competition with elastic subscription demand.[6] In particular, we explain how rational consumer expectations are formed. Expectations about network size are important since consumers care about the size of each network when firms charge different prices for on and off-net calls. In section 3 we characterize the unique symmetric equilibrium in case of a fixed per minute termination charge close to cost. In order to disentangle the business stealing effect from the network externality effect, we also analyze a model of two interconnected monopolistic networks. In section 4 we introduce the retail benchmarking approach and show it outperforms any rule based on fixed (per minute) termination charges. In section 5 we show that, if the regulator has the same information as the networks, a minor modification of the benchmarking rule even induces the first best outcome, where all prices equal cost and market penetration is maximized. Section 6 concludes.

2 The model

2.1 The Logit Model

We consider competition between two networks. Each firm i ($i = 1, 2$) charges a fixed fee F_i and may discriminate between on-net and off-net calls. Firm i's marginal on-net price is written p_i and off-net price is written $\hat{p}_i$. The total mass of consumers is normalized to 1. Consumer's utility from making calls of length q is given by a concave and increasing utility function $u(q)$. Demand $q(p)$ is defined by $u'(q(p)) = p$. The marginal cost of a call equals c and the termination cost equals $c_0(\leq c)$. The reciprocal access charge is denoted a. We

[6] For an introduction of logit models see Anderson and de Palma (1992) and Anderson, de Palma and Thisse (1992).

define $v(p) = u(q(p)) - pq(p)$. Note that $v'(p) = -q(p)$.

The timing of the game is as follows:

First, firms $i = 1, 2$ choose simultaneously tariffs $T_i = (F_i, p_i, \hat{p}_i)$.

Next, consumers form expectations about the number of subscribers of network 1 (β_1) and network 2 (β_2), with $\beta_i \geq 0$ and $\beta_1 + \beta_2 \leq 1$. We assume that all consumers have the same expectations (and these are in principle functions of the tariffs). Given such expectations, utility from subscribing to network 1 equals

$$V_1 = \beta_1 v(p_1) + \beta_2 v(\hat{p}_1) - F_1,$$

while subscribing to network 2 yields

$$V_2 = \beta_2 v(p_2) + \beta_1 v(\hat{p}_2) - F_2.$$

Finally, not subscribing at all yields utility V_0. Define $U_1 = V_1 + \mu\varepsilon_1$, $U_2 = V_2 + \mu\varepsilon_2$, and $U_0 = V_0 + \mu\varepsilon_0$.

The parameter $\mu > 0$ reflects the degree of product differentiation. A high value of μ implies that most of the value is determined by a random draw so that competition between the firms is rather weak. The noise terms ε_k are random variables of zero mean and unit variance, identically and independently double exponentially distributed. They reflect consumers' preference for one good over another. A consumer will subscribe to network 1 if and only if $U_1 > U_2$ and $U_1 > U_0$; he will subscribe to network 2 if and only if $U_2 > U_1$ and $U_2 > U_0$; he will not subscribe to any network otherwise. These probabilities are denoted by $\wp_k$. These probabilities are given by

$$\wp_1 = \frac{\exp[V_1/\mu]}{\sum_{k=0}^{2} \exp[V_k/\mu]},$$

$$\wp_2 = \frac{\exp[V_2/\mu]}{\sum_{k=0}^{2} \exp[V_k/\mu]},$$

$$\wp_0 = \frac{\exp[V_0/\mu]}{\sum_{k=0}^{2} \exp[V_k/\mu]}.$$

2.2 Rational Expectations

For expectations to be rational, we need $\beta_i = \wp_i$. For any price schedules T_1, T_2 such self-fulfilling expectations exist as these are fixed points of the mapping $f : \Delta^2 \to \Delta^2$ where

$f(\beta) = (\wp_1, \wp_2)$. The fixed point is unique as long as μ is sufficiently high. This can be shown by looking at the index of zeros of the mapping $g(\beta) = f(\beta) - \beta$. The Jacobian of g is

$$D_\beta g = \begin{pmatrix} \frac{1}{\mu}[\beta_1(1-\beta_1)v(p_1) - \beta_1\beta_2 v(\hat{p}_2)] - 1 & \frac{1}{\mu}[\beta_1(1-\beta_1)v(\hat{p}_1) - \beta_1\beta_2 v(p_2)] \\ \frac{1}{\mu}[\beta_2(1-\beta_2)v(\hat{p}_2) - \beta_1\beta_2 v(p_1)] & \frac{1}{\mu}[\beta_2(1-\beta_2)v(p_2) - \beta_1\beta_2 v(\hat{p}_1)] - 1 \end{pmatrix}.$$

Let $d = \det D_\beta g$. For large enough μ, $d > 0$ so that the Jacobian is non-singular. This implies that for large enough μ rational expectations are uniquely defined.

Proposition 1 *For any pricing schedules (T_1, T_2) rational expectations exist. If $\det D_\beta g > 0$ for any rational expectation, rational expectations are uniquely defined.*

A necessary condition for rational expectations to be unique is that the determinant is positive at the symmetric equilibrium candidate $(p, \hat{p}, F)$ with per firm subscribers equal to $\wp \leq 1/2$. Let $v = v(p)$ and $\hat{v} = v(\hat{p})$. The determinant can be rewritten in this case to be equal to

$$d = (\mu^2 + 2\mu\wp((\wp - 1)v + \wp\hat{v}) + \wp^2(1 - 2\wp)(v^2 - \hat{v}^2))/\mu^2,$$

which can in turn be rewritten as

$$d = [\mu - \wp(v - \hat{v})][\mu - \wp(v + \hat{v})(1 - 2\wp)]/\mu^2 \tag{1}$$

which is strictly positive (given that $\mu > 0$) if and only if $\mu > \wp(v + \hat{v})(1 - 2\wp)$ and $\mu > \wp(v - \hat{v})$. In particular, this will be satisfied for *any* $\wp \in [0, 1/2]$ when $\hat{v} \approx v$ and $\mu > v/4$. It is also satisfied for smaller values of μ as long as $v \approx \hat{v}$ and $\wp \approx 1/2$ or $\wp \approx 0$. Also note that if $p_1 = p_2 = \hat{p}_1 = \hat{p}_2 = c$, but possibly $F_1 \neq F_2$ (and thus $\beta_1 \neq \beta_2$) then

$$d = \frac{\mu - (1 - \beta_1 - \beta_2)(\beta_1 + \beta_2)v}{\mu} > 0$$

where the inequality follows when $\mu > v/4$.

If expectations are not uniquely defined, one can potentially construct many equilibria by having even the tiniest of deviations lead to expectations that jump, in the direction that makes such deviations unprofitable. To avoid the existence of equilibria supported by such jumping expectations, we will henceforth assume that $\mu > v(c)/4$, where $v(c)$ equals the surplus when calls are priced at marginal cost. This is sufficient and necessary for unique rational expectations to exist when access charge is close to c_0.

Assumption 1 $\mu > v(c)/4$.

Note that

$$D_{F_1} g = \begin{pmatrix} -\beta_1(1-\beta_1)/\mu \\ \beta_1\beta_2/\mu \end{pmatrix}.$$

This implies that an increase in the fixed fee of network 1, everything else equal, will decrease the number of subscribers to network 1 and will increase the subscribers to network 2. However, a change in F_1 also affects expectations and the total effect on the number of subscribers by a change in fixed fee F_1 is given by the implicit function theorem as

$$D_{F_1}\beta = -[D_\beta g]^{-1} D_{F_1} g.$$

One thus verifies that

$$\frac{\partial \beta_1}{\partial F_1} = \frac{-1}{d\mu^2}\beta_1((1-\beta_1)\mu - \beta_2(1-\beta_1-\beta_2)v(p_2)) \tag{2}$$

and

$$\frac{\partial \beta_2}{\partial F_1} = \frac{-1}{d\mu^2}\beta_1\beta_2(v(\hat{p}_2)(1-\beta_1-\beta_2) - \mu). \tag{3}$$

A special case of interest will be where both firms charge marginal cost c for on-net and off-net calls, i.e., $p_1 = p_2 = \hat{p}_1 = \hat{p}_2 = c$. In this case we have

$$\frac{\partial \beta_1}{\partial F_1} = \frac{-\beta_1((1-\beta_1)\mu - \beta_2(1-\beta_1-\beta_2)v(c))}{\mu - v(c)(\beta_1+\beta_2)(1-\beta_1-\beta_2)} \tag{4}$$

and

$$\frac{\partial \beta_2}{\partial F_1} = \frac{\beta_1\beta_2(\mu - v(c)(1-\beta_1-\beta_2))}{\mu - v(c)(\beta_1+\beta_2)(1-\beta_1-\beta_2)}. \tag{5}$$

Summing up the previous equations, one obtains

$$\frac{\partial(\beta_1+\beta_2)}{\partial F_1} = \frac{-\beta_1(1-\beta_1-\beta_2)}{\mu - v(c)(\beta_1+\beta_2)(1-\beta_1-\beta_2)} < 0.$$

Market penetration thus decreases when one firm increases its fixed fee. The firm increasing its fixed fee loses subscribers. Whether the rival firm loses or gains subscribers depends on the sign of $\Delta = \mu - v(c)\beta_0$ where $\beta_0 = 1-\beta_1-\beta_2$ denotes the number of unsubscribed consumers. If $\Delta > 0$, the rival firm gets more subscribers while if $\Delta < 0$, it gets less subscribers. For instance, in the extreme case of full subscription (i.e. $\beta_1 + \beta_2 = 1$), $\Delta = \mu > 0$ since there is only a business stealing effect: all consumers who leave firm 1 subscribe to firm 2. In

contrast, when subscription is voluntary, an increase in the fixed fee of firm 1 will lead to some consumers switching to firm 2 and some consumers becoming unsubscribed. If consumers do not immediately adjust their expectations, the proportion of all the consumers that leave firm 1, and then go to firm 2 (rather than becoming unsubscribed), is proportional to β_2/β_0. Once consumers realize that the value of being subscribed is reduced because there are more unsubscribed consumers, some of the subscribers of firm 2 will become unsubscribed. So firm 2 gains some subscribers due to the business stealing effect, but also loses some subscribers due to the network externality effect. Clearly, if β_0 is relatively large, a relatively large fraction of consumers leaving firm 1 will become unsubscribed, so that the network externality effect becomes large. In order to see whether the net effect is positive or negative, note that, in the Logit model $\log(\beta_2) - \log(\beta_0) = (V_2 - V_0)/\mu$. This implies that (when all usage prices are c)

$$\beta_2 = \beta_0 \exp[((1-\beta_0)v - F_2 - V_0)/\mu].$$

The derivative of the right-hand side with respect to β_0 equals $(1 + \beta_0(-v/\mu)) \exp[((1 - \beta_0)v - F_2 - V_0)/\mu]$, which is positive if and only if $\Delta = \mu - (1 - \beta_1 - \beta_2)v > 0$. We will say that there is *a net business stealing effect* if $\Delta > 0$ and there is *a net network externality effect* if $\Delta < 0$.

At a symmetric equilibrium candidate $\beta_1 = \beta_2 = \wp$ we have

$$\frac{\partial \wp_1}{\partial F_1} = \frac{-\wp}{d\mu^2}\left[(1-\wp)\mu - \wp(1-2\wp)v\right],$$

and

$$\frac{\partial \wp_2}{\partial F_1} = \frac{-\wp^2}{d\mu^2}\left[\hat{v}(1-2\wp) - \mu\right].$$

It will be convenient for later purposes if we already denote the effect on the total number of subscribers with respect to an increase in the fixed fee of one of the networks, at the symmetric equilibrium candidate:

$$\frac{\partial \wp_1}{\partial F_1} + \frac{\partial \wp_2}{\partial F_1} = \frac{\wp(-1+2\wp)}{\mu - \wp(1-2\wp)(v+\hat{v})} < 0. \tag{6}$$

In a symmetric equilibrium candidate $T = (p, \hat{p}, F)$, rational expectations thus require the following relation between fees and number of subscribers per firm:

$$\wp = \frac{\exp[(\wp(v(p) + v(\hat{p})) - F)/\mu]}{2\exp[(\wp(v(p) + v(\hat{p})) - F)/\mu] + \exp[V_0/\mu]}. \tag{7}$$

3 Competition with fixed per minute termination charge

Let us now consider the case with a constant reciprocal access fee a. Write $R(p) = (p-c)q(p)$. Note that the number of subscribers $\wp_i$ and $\wp_j$ depend on V_0 and tariff schedule T_1, T_2. We will omit arguments. Profit can be written as follows

$$\Pi_i = \wp_i[\wp_i R(p_i) + \wp_j R(\hat{p}_i) + F_i - f] + \wp_i \wp_j (a - c_0)(q(\hat{p}_j) - q(\hat{p}_i)). \tag{8}$$

Firm i maximizes profits by setting T_i, holding T_j constant. Note that a change in marginal price p_i or $\hat{p}_i$ while holding F_i fixed will affect the number of subscribers to i as well as of j. For example, a decrease in price will make network i more attractive and will thus attract some subscribers of j and will also attract some consumers who previously did not subscribe to any network. This then makes it also more attractive to subscribe to network j relative to staying unsubscribed, because of the network effect. It will be convenient to apply a change of variables and let network i maximize profits by choosing p_i, $\hat{p}_i$, and $\wp_i$, holding p_j, $\hat{p}_j$ and F_j fixed. This can be done since there is a monotonic relationship between F_i and $\wp_i$ since $\partial \wp_i / \partial F_i < 0$.

Note that, at the fixed point, one has

$$\frac{\wp_i}{1 - \wp_i} = \frac{\exp\left[V_i/\mu\right]}{\exp[V_j/\mu] + \exp[V_0/\mu]},$$

so that

$$F_i = \wp_i v(p_i) + \wp_j v(\hat{p}_i) - \mu \log \left[\frac{\wp_i}{1 - \wp_i} (\exp[V_j/\mu] + \exp[V_0/\mu]) \right].$$

Holding everything but p_i and F_i fixed, one obtains

$$\partial F_i / \partial p_i = \wp_i v'(p_i).$$

Similarly, holding everything but $\hat{p}_i$ and F_i fixed, one obtains

$$\partial F_i / \partial \hat{p}_i = \wp_j v'(\hat{p}_i).$$

Note that if p_i is changed while keeping $\wp_i$, $\hat{p}_i$, p_j, $\hat{p}_j$ and F_j fixed, then also $\wp_j$ will remain fixed. Maximizing with respect to on-net price p_i (keeping $\wp_i$ fixed) thus yields

$$0 = \frac{\partial \Pi_i}{\partial p_i} = \wp_i^2 (R'(p_i) + v'(p_i)) = \wp_i^2 (p_i - c) q'(p_i).$$

Hence, $p_i = c$. In words, on-net calls are priced at marginal cost.

Maximizing with respect to off-net price $\hat{p}_i$ (keeping $\wp_i$ fixed) yields

$$0 = \frac{\partial \Pi_i}{\partial \hat{p}_i} = \wp_i \wp_j (R'(\hat{p}_i) + v'(\hat{p}_i) - (a - c_0) q'(\hat{p}_i)) = \wp_i \wp_j (\hat{p}_i - c - a + c_0) q'(\hat{p}_i).$$

Hence, $\hat{p}_i = c + a - c_0$. In words, off-net calls are priced at perceived marginal cost (i.e. the off-net marginal cost).

We thus obtain the standard "perceived" marginal cost pricing result under non-linear pricing. Given $p_1 = p_2 = c$ and $\hat{p}_1 = \hat{p}_2 = \hat{c}$, profits can be rewritten as

$$\Pi_i = \wp_i [\wp_j R(\hat{c}) + F_i - f].$$

Thus

$$\frac{\partial \Pi_i}{\partial F_i} = \frac{\partial \wp_i}{\partial F_i} [\wp_j R(\hat{c}) + F_i - f] + \wp_i [\frac{\partial \wp_j}{\partial F_i} R(\hat{c}) + 1].$$

So the first order condition reads

$$0 = \frac{\partial \wp_i}{\partial F_i} (F_i - f) + \wp_i + R(\hat{c}) \left(\wp_j \frac{\partial \wp_i}{\partial F_i} + \wp_i \frac{\partial \wp_j}{\partial F_i} \right).$$

Solving for a symmetric solution, and using the marginal effects on the number of subscribers of networks 1 and 2 with respect to a change in the fixed fee of network 1 (equation (6)), yields

$$F - f = \frac{-\wp - R(\hat{c}) \frac{\wp^2(-1+2\wp)}{\mu - \wp(1-2\wp)(v+\hat{v})}}{\frac{\partial \wp_i}{\partial F_i}}.$$

This can be manipulated to yield

$$F = f + \frac{\mu - (1-2\wp)\wp(v + \hat{v} + R(\hat{c}))}{(1-\wp)\mu - \wp(1-2\wp)v} (\mu - \wp(v - \hat{v})) =: F^{equil}(\wp, a) \tag{9}$$

On the other hand, rational expectations, by means of equation (7) determines the following relation between symmetric equilibrium fixed fee and equilibrium subscription:

$$F = \wp(v + \hat{v}) - V_0 - \mu \ln \left[\frac{\wp}{1 - 2\wp} \right] =: F^{RE}(\wp, a). \tag{10}$$

The equilibrium number of subscribers per firm is thus found by solving $F^{equil}(\wp, a) = F^{RE}(\wp, a)$. We will denote this solution by $\wp(a)$. In particular, for $a = c_0$ the solution is

given by

$$\left[f+\mu\frac{\mu-2\wp(1-2\wp)v}{(1-\wp)\mu-\wp(1-2\wp)v}\right]-\left[2\wp v-V_0-\mu\ln\left(\frac{\wp}{1-2\wp}\right)\right]=0.$$

It can be shown (using $\mu/v > 1/4$) that there is a unique solution $\wp^* = \wp(c_0)$ to this equation. There will then also be a unique solution for $a \neq c_0$ for small enough $|a - c_0|$. Moreover, $F^{RE}(\wp, c_0) > F^{equil}(\wp, c_0)$ if and only if $\wp < \wp^*$. That is, the rational expectations curve cuts the equilibrium curve from above.

Proposition 2 *For $|a - c_0|$ small enough, there exists a unique symmetric equilibrium $(p, \hat{p}, F)$. This solution is given by $p = c$, $\hat{p} = \hat{c}$ and $F = F^{RE}(\wp(a), a)$.*

We will be particularly interested in how profits, consumer surplus and total surplus vary with a. It turns out that analyzing these effects is not straightforward since there are two opposing effects at work. On the one hand, firms would want to use the termination charge to soften price competition to raise fixed fees and profits. This is the only force at work in Gans and King (2001) where subscription demand is inelastic. However, in the Logit model with elastic subscription demand there is a second force at work, namely network externalities. Firms may have a common incentive to increase market penetration as this increases the value of subscription to each customer. It is not obvious which of the two effects dominates. Moreover, in the case the network externality effect dominates, it is also not clear whether firms or the regulator would like to increase or decrease the termination charge. Therefore, we analyze in the next subsection a model where only the network externality effect exists.

3.1 Two islands: the case of two interconnected monopolists

In order to provide more insight and intuition about how different access prices affect the possibility to internalize network effects and how it affects competition, we present in this section a model where firms do not compete with each other. There are two islands and firm i $(= 1, 2)$ operates only in island i. Each island has a population normalized to $1/2$. Inhabitants of an island cannot (or simply do not want to) subscribe to the operator of the other island. Hence, the two firms do not compete for the same customers. However, the inhabitants care indirectly about the pricing policy of the "monopolist" on the other island since it affects subscription rates and thus affects how many calls can be made to its subscribers.

As before, consumers form expectations over the number of subscribers of network 1 (β_1) and network 2 (β_2), with $\beta_i \geq 0$ and $\beta_1 + \beta_2 \leq 1$. Given such expectations, utility from subscribing to network 1 (for inhabitants of island 1) equals

$$V_1 = \beta_1 v(p_1) + \beta_2 v(\hat{p}_1) - F_1,$$

while subscribing to network 2 (for inhabitants of island 2) yields

$$V_2 = \beta_2 v(p_2) + \beta_1 v(\hat{p}_2) - F_2.$$

Finally, not subscribing at all yields utility V_0. Define $U_1 = V_1 + \mu\varepsilon_1$, $U_2 = V_2 + \mu\varepsilon_2$, and $U_0 = V_0 + \mu\varepsilon_0$. Consumers from island i subscribe (to network i) when $U_i > U_0$ and remain unsubscribed otherwise.

V_1 and V_2 are as before but now the number of subscribers on island i equals

$$\wp_i = \frac{1}{2} \times \frac{\exp[V_i/\mu]}{\exp[V_i/\mu] + \exp[V_0/\mu]}.$$

Rational expectations imply $\beta_i = \wp_i$. These are zeros of the mapping $\tilde{g}(\beta) = (\wp_1 - \beta_1, \wp_2 - \beta_2)$. The Jacobian of g is now

$$D_\beta \tilde{g} = \begin{pmatrix} \frac{1}{\mu}[\beta_1(1-2\beta_1)v(p_1)] - 1 & \frac{1}{\mu}[\beta_1(1-2\beta_1)v(\hat{p}_1)] \\ \frac{1}{\mu}[\beta_2(1-2\beta_2)v(\hat{p}_2)] & \frac{1}{\mu}[\beta_2(1-2\beta_2)v(p_2)] - 1 \end{pmatrix}.$$

Let $\tilde{d}$ denote the determinant of this Jacobian at a symmetric solution $b = \beta_1 = \beta_2$. Then

$$\tilde{d} = [\mu - b(1-2b)(v+\hat{v})][\mu - b(1-2b)(v-\hat{v})]/\mu^2.$$

For large enough μ we have $\tilde{d} > 0$ so then rational self-fulfilling expectations are unique.

Note that

$$D_{F_1}\tilde{g} = \begin{pmatrix} -\beta_1(1-2\beta_1)/\mu \\ 0 \end{pmatrix}.$$

This implies that an increase in the fixed fee of network 1, everything else equal, will decrease the number of subscribers to network 1 and will keep the number of subscribers to network 2 constant. The latter illustrates the fact that there is no business stealing effect in this model. However, a change in F_1 also affects expectations and the total effect on the number

of subscribers by a change in fixed fee F_1 is given by the implicit function theorem as

$$D_{F_1}\beta(F_1) = -[D_\beta \tilde{g}]^{-1} D_{F_1}\tilde{g}.$$

One thus verifies that

$$\frac{\partial \beta_1}{\partial F_1} = \frac{-1}{d\mu^2}\beta_1((1-2\beta_1)(\mu - \beta_2(1-2\beta_2)v)$$

and

$$\frac{\partial \beta_2}{\partial F_1} = \frac{-1}{d\mu^2}\beta_1\beta_2(1-2\beta_1)(1-2\beta_2)\hat{v}.$$

Thus, at a symmetric solution $\beta_1 = \beta_2 = \beta$

$$\frac{\partial \beta_1}{\partial F_1} = \frac{-1}{d\mu^2}\beta((1-2\beta)(\mu - \beta(1-2\beta)v) < 0$$

and

$$\frac{\partial \beta_2}{\partial F_1} = \frac{-1}{d\mu^2}\beta^2(1-2\beta)^2\hat{v} < 0.$$

Note that an increase of the fixed fee of firm 1 results in a decrease of the number of subscribers of firm 2. As before, firms will set variable price equal to perceived marginal cost. Given $p_1 = p_2 = c$ and $\hat{p}_1 = \hat{p}_2 = \hat{c} := c + a - c_0$, profits can be rewritten as

$$\Pi_i = \wp_i[\wp_j R(\hat{c}) + F_i - f].$$

So the first order condition reads

$$0 = \frac{\partial \wp_i}{\partial F_i}(F_i - f) + \wp_i + R(\hat{c})\left(\wp_j \frac{\partial \wp_i}{\partial F_i} + \wp_i \frac{\partial \wp_j}{\partial F_i}\right).$$

Solving for a symmetric solution, and using the marginal effects on the number of subscribers of networks 1 and 2 with respect to a change in the fixed fee of network 1, yields

$$F - f = \frac{-\wp - R(\hat{c})\frac{-\wp^2(1-2\wp)}{\mu - \wp(1-2\wp)(v+\hat{v})}}{\frac{\partial \wp_i}{\partial F_i}}.$$

This can be manipulated to yield

$$F = f + \frac{\mu - (1-2\wp)\wp(v + \hat{v} + R(\hat{c}))}{(1-2\wp)(\mu - \wp(1-2\wp)v)}(\mu - \wp(1-2\wp)(v - \hat{v})) \tag{11}$$

It is readily verified that the righthand-side of this equation is decreasing in a at $a = c_0$:

$$\frac{\partial RHS(11)}{\partial a}|_{a=c_0} = -\wp q(c)\frac{\mu - 2\wp(1-2\wp)v}{\mu - \wp(1-2\wp)v}.$$

To have rational expectations fulfilled we have

$$F = \wp(v + \hat{v}) - W_0 - \mu \ln\left[\frac{2\wp}{1-2\wp}\right]. \tag{12}$$

Note that, at $a = c_0$, the righthand-side of this equation is decreasing in a:

$$\frac{\partial RHS(12)}{\partial a}|_{a=c_0} = -\wp q(c).$$

Hence, a marginal increase of a above c_0 makes the rational expectations curve drop by more than the equilibrium condition curve. This means that the number of subscribers goes down when a is increased above c_0.

Lemma 1 *In the Logit model with two interconnected monopolists, an increase in the termination charge above c_0 lowers overall market penetration.*

So if firms want to increase market penetration, they will want termination charge below cost. The intuition is that firms realize that raising ones fixed fee reduces the size of the other networks and thus hurts its own customers. However, they fail to internalize the fact that this also hurts the other network, and therefore they set a too high fixed fee. By having $a < c_0$, the value of making off-net calls is higher. This means that subscribers from the second network care more about the size of the first network. An increase in the fixed fee of network 1 will now thus reduce the size of the other network more than when $a = c_0$. Hence, letting $a < c_0$ exacerbates the negative effect of raising ones fee on its own subscribers, and firms thus lower the fixed fee and increase market penetration.

Do firms really want to increase market penetration? How does it affect profit and welfare? To that end let us first investigate how profit is affected by a marginal increase in $\wp$ (the number of subscribers of each network) above the equilibrium value $\wp^*$ (when a is fixed at c_0). Rational expectations have to be met, which means we are considering how profits vary along the rational expectations curve. Note that at the equilibrium (at $a = c_0$) per consumer profit equals

$$F - f = \frac{\mu(\mu - 2\wp^*(1-2\wp^*)v}{(1-2\wp^*)(\mu - \wp^*(1-2\wp^*)v)}.$$

The effect on total profit $H(c_0, \wp) = \wp\,(F - f)$ with respect to a change in $\wp$ is thus

$$\begin{aligned}\frac{\partial H}{\partial \wp}(c_0, \wp^*) &= H(c_0, \wp^*)/\wp^* + \wp^* \left[2v(c) - \frac{\mu}{\wp^*(1 - 2\wp^*)}\right] \\ &= \frac{\wp^* v(\mu - 2\wp^*(1 - 2\wp^*)v)}{\mu - \wp^*(1 - 2\wp^*)v} > 0\end{aligned}$$

Hence, profits increase along the rational expectations curve, in a neighborhood around $\wp^*$.

Next, we have to account for the fact that when a is varied, the rational expectations curve, and thus the equilibrium, will change. The partial effect on profits (keeping $\wp^*$ fixed) equals

$$\frac{\partial H}{\partial a} = (\wp^*)^2(a - c_0)q'(\hat{c}),$$

so that at $a = c_0$ a marginal change in a does not affect profits directly. The extra profit made through access revenues is just offset by the decrease in the fixed fee. However, profits are affected indirectly by a change in market penetration.

$$\frac{dH}{da} = H_a + H_\wp \times \wp'(a) < 0.$$

One observes that profits are decreasing in a in a neighborhood around c_0. Firms thus indeed prefer an access fee below cost. This leads to higher market penetration which means it is also good for consumer and total welfare.

Proposition 3 *In the logit model with two interconnected monopolists, firms prefer access fee $a < c_0$. This also improves consumer welfare.*

To provide the intuition for the result, it is useful to note that the two monopolists would like to achieve the outcome as close as possible to the outcome that would be chosen by a monopolist operating in both islands. Given that the two monopolists do not fully internalize network externalities, the number of subscribers is smaller than the one under a monopolist operating in both islands. Therefore, they want to increase the subscribers by choosing an access charge below the termination cost.

3.2 Interconnected duopoly

We now return to the case of competing interconnected duopolists. As explained before, the case of interconnected duopolist exhibits both network externalities and business stealing effects. We here analyze the effect of a change in termination charge around c_0 for profits, consumer welfare and total welfare.

We first analyze how an increase in a effects market penetration. Let us define $h(\wp, a) = F^{equil}(\wp, a) - F^{RE}(\wp, a)$.

$$h(\wp, a) = \frac{\mu - (1-2\wp)\wp(v + \hat{v} + R(\hat{c}))}{(1-\wp)\mu - \wp(1-2\wp)v}(\mu - \wp(v - \hat{v})) - \wp(v + \hat{v}) + W_0 + f + \mu \ln\left[\frac{\wp}{1-2\wp}\right].$$

We have already established that there is a unique solution $\wp(a)$ of $h(\wp, a) = 0$. Moreover, at the solution $h_\wp > 0$. Hence,

$$\wp'(a) = -\frac{h_a}{h_\wp}$$

has the opposite sign as h_a.

$$\frac{\partial h}{\partial a}(\wp, c_0) = \frac{\wp^2 q(c)}{(1-\wp)\mu - \wp(1-2\wp)v}((1-2\wp)v - \mu).$$

We conclude that for $\Delta^* = \mu - (1-2\wp^*)v > 0$ an increase in a above c_0 will increase the equilibrium number of subscribers, while for $\Delta^* < 0$ such an increase in a results in a decrease in the equilibrium number of subscribers.

Lemma 2 *Let* $\Delta^* = \mu - (1-2\wp^*)v$.

$$\frac{d\wp^*(a)}{da}|_{a=c_0} > 0 \textit{ if and only if } \Delta^* > 0$$

and

$$\frac{d\wp^*(a)}{da}|_{a=c_0} < 0 \textit{ if and only if } \Delta^* < 0.$$

If $\Delta^* < 0$, then $\partial\wp_2/\partial F_1 < 0$. This means that an increase in network 1's fixed fee reduces the number of subscribers to network 2. In this case the network effect dominates the business stealing effect. One would expect that firms then would prefer termination charge below cost in order to boost market penetration. If $\Delta^* > 0$ the business stealing effect dominates and one would expect firms again to prefer termination charge below cost, in this case to reduce market penetration and to increase fixed fees and profits. We now proceed to verify that indeed firms always prefer below cost termination charges, although for different reasons.

Let $H(a, \wp)$ denote the profit a firm makes when it has $\wp$ subscribers, access fee is a and the fixed fee is given by $F^{RE}(\wp, c_0)$. That is

$$H(a, \wp) = \wp(\wp R(\hat{c}) + F - f) = \wp(\wp(R(\hat{c}) + v(c) + v(\hat{c})) - W_0 - \mu \log[\wp/(1-2\wp)] - f).$$

We will be interested in knowing what happens with this profit at $a = c_0$ when $\wp$ is moved away from the corresponding equilibrium value $\wp^*$. Note that we know that per consumer profit at the equilibrium equals $F - f$, which by (9) equals (at $a = c_0$)

$$\mu \frac{\mu - 2\wp^*(1 - 2\wp^*)v}{(1 - \wp^*)\mu - \wp^*(1 - 2\wp^*)v}.$$

Hence,

$$\begin{aligned}
\frac{\partial H}{\partial \wp}(c_0, \wp^*) &= H(c_0, \wp^*)/\wp^* + \wp^* \left[2v(c) - \frac{\mu}{\wp^*(1 - 2\wp^*)} \right] \\
&= \mu \frac{\mu - 2\wp^*(1 - 2\wp^*)v}{(1 - \wp^*)\mu - \wp^*(1 - 2\wp^*)v} + 2\wp^* v - \frac{\mu}{1 - 2\wp^*} \\
&= \frac{-\wp^*(\mu - (1 - 2\wp^*)v)(\mu - 2\wp^*(1 - 2\wp^*)v)}{(1 - 2\wp^*)((1 - \wp^*)\mu - \wp^*(1 - 2\wp^*)v)}
\end{aligned}$$

Hence, the sign of this derivative is opposite to the sign of $\Delta = \mu - (1 - 2\wp^*)v$. Thus, if an increase of a above c_0 increases market penetration, profits decrease with the number of subscribers *along* the rational expectations curve. And if an increase of a above c_0 decreases market penetration, then profits increase with the number of subscribers along the rational expectations curve, in a neighborhood around c_0.

Next, we have to account for the fact that when a is varied, the rational expectations curve, and thus the equilibrium, will change. The partial effect on profits (keeping $\wp^*$ fixed) equals

$$\frac{\partial H}{\partial a} = (\wp^*)^2 (a - c_0) q'(\hat{c}),$$

so that at $a = c_0$ a marginal change in a does not affect profits directly. The extra profit made through access revenues is just offset by the decrease in fixed fee. However, profits are affected indirectly by a change in market penetration.

$$\frac{dH}{da} = H_a + H_\wp \times \wp'(a).$$

Since the sign of $H_\wp$ is the opposite of the sign of $\wp'(a)$ at $a = c_0$, one observes that profits are always decreasing in a in a neighborhood around c_0. Firms thus always prefer an access fee below cost.

Proposition 4 *Firms prefer access fee $a < c_0$.*

How does total surplus change when the access fee is changed? Total surplus is the sum of consumer surplus and industry profit. The expression for consumer surplus in the Logit

model has been derived by Small and Rosen (1981) as (up to a constant)

$$CS(a, \wp(a)) = \mu \ln \left(\sum_{j=0}^{2} \exp(V_j/\mu) \right) = V_0 - \mu \ln(1 - 2\wp(a)).$$

Hence, consumer surplus is not directly affected by the access charge, but only through the equilibrium number of subscribers. Clearly, consumer surplus is increasing in the number of subscribers:

$$\frac{\partial\, CS}{\partial \wp} = \frac{2\mu}{1 - 2\wp} > 0.$$

$$TS(a, \wp(a)) = CS(a, \wp(a)) + 2H(a, \wp(a)).$$

We thus obtain, at $a = c_0$,

$$\begin{aligned} \frac{d\, TS}{d\, a} &= \wp'(a) CS_\wp + 2H_a + 2H_\wp \wp'(a) \\ &= \wp'(c_0) \left(\frac{2\mu}{1 - 2\wp} + 2 \frac{-\wp(\mu - (1 - 2\wp)v)(\mu - 2\wp(1 - 2\wp)v)}{(1 - 2\wp)((1 - \wp)\mu - \wp(1 - 2\wp)v)} \right) \end{aligned}$$

It can be established that the term in brackets is strictly positive when $\mu > v/4$. This means that total surplus is increased when market penetration is increased.

Proposition 5 *Let $\wp^*$ denote the number of subscribers per firm in the equilibrium when $a = c_0$. If $\mu > (1 - 2\wp^*)v$, the number of subscribers, and thus total surplus, increases as a is increased above c_0. If $\mu < (1 - 2\wp^*)v$, the number of subscribers, and thus total surplus, increases as a is decreased below c_0: in this case, the socially desirable access fee is lower than the fee that maximizes industry profit.*

Both the firms and the social planner want to divert from the access charge equal to termination cost in order to affect the number of subscribers. The firms want to make the number of subscribers as close as possible to the number chosen by a monopolist owning both networks while the social planner always wants to increase the number. When there is a net business-stealing effect (i.e. $\Delta^* > 0$), there is a conflict of interest between the firms and the social planner since the firms want to decrease the number of subscribers, which requires them to choose a below c_0 in order to soften competition. When there is a net network externality effect (i.e. $\Delta^* < 0$), there is a congruence of interests between the firms and the social planner in the sense that firms want to increase the number of subscribers, which again requires them to choose a below c_0 in order to internalize network externalities.

However, the firms do not internalize the positive effect that an increase in network has on consumer surplus and therefore the socially preferred access charge is lower than the one preferred by the firms.

4 Retail Benchmarking Approach

In this section, we generalize the retail benchmarking approach introduced in Jeon and Hurkens (2008). Jeon and Hurkens (2008) consider the case without termination-based price discrimination and with full participation of all consumers and find a class of access pricing rules parameterized by κ that achieves the marginal cost pricing (i.e. the call charge equal to c). We generalize the previous result in three dimensions. First, we allow for termination-based price discrimination. Second, we consider a Logit model with elastic subscription demand where full participation never arises. Third, in this setting, for a given fixed access charge a, we find a class of access pricing rules parameterized by κ that induces each network to choose the on-net price equal to the on-net marginal cost and the off-net price equal to the off-net marginal cost.

Before generalizing the retail benchmarking approach, we remind the regulator's information constraint and the result from Jeon and Hurkens (2008).

4.1 Assumption and Result from Jeon and Hurkens (2008)

We maintain the same constraint on the regulator's information as in our previous paper:

• *The regulator's informational constraint*: On the one hand, we assume that the regulator has limited information about the market such that she is not informed about the individual demand function $q(p)$, each firm's subscription demand function and the value of the fixed cost f. On the other hand, she knows the marginal cost c and the termination cost c_0. Furthermore, she and consumers observe retail prices $[(p_1, \widehat{p}_1, F_1), (p_2, \widehat{p}_2, F_2)]$. Moreover, we need to assume that the regulator can observe average retail prices,[7] which means that she is able to observe realized demand.

The firms are assumed to know all the relevant information regarding both the demand and the cost sides as in Jeon and Hurkens (2008).

In a model without termination-based price discrimination and with full participation

[7]For instance, the Spanish telecommunication agency (Comisión del Mercado de las Telecomunicaciones) publishes data on each network's average price.

of all consumers, we find that the following family of access pricing rules parameterized by $\kappa(< 1)$ induces each firm to adopt the marginal cost pricing (i.e. $p_i = c$):

$$a_i = c_0 + \kappa \left(\frac{F_i + p_i q(p_i)}{q(p_i)} - c \right), \tag{13}$$

where a_i represents the access charge that firm i pays to each rival firm. $\kappa = 0$ corresponds to the fixed access charge equal to the termination cot. According to the rule, the mark-up of the access charge that firm i pays to each rival firm is equal to the firm i's average price mark-up multiplied by κ. We find that the retail benchmarking rule intensifies retail competition such that higher values of κ translate into lower fixed fees. However, that does not affect total surplus when all consumers subscribe to one of the two networks, as we assumed in our previous paper.

4.2 Generalization

Consider a given fixed and reciprocal access charge a that can be different from c_0. Let $\pi_i(a)$ denote network i's retail profit per customer gross of the fixed cost;

$$\pi_i(a) \equiv \wp_i(p_i - c)q(p_i) + \wp_j(\widehat{p}_i - (c + a - c_0))q(\widehat{p}_i) + F_i$$

Therefore, given the fixed access charge a, network i's profit is given by

$$\Pi_i = \wp_i \left[\pi_i(a) + \wp_j(a - c_0)q(\widehat{p}_j) - f \right]$$

We remind from the previous sections that in this case, network i finds it optimal to choose $p_i = c$ and $\widehat{p}_i = c + a - c_0$.

We now propose the following generalization of our access pricing rule: *in addition to paying the fixed (per-minute) access charge a,* network i pays an access charge a_i determined by

$$a_i q(\widehat{p}_i) = \kappa \pi_i(a).$$

so that total termination charge equals

$$\lambda(a, \kappa) := a + \kappa \frac{\pi_i(a)}{q(\widehat{p}_i)}. \tag{14}$$

Under our generalized access pricing rule, network i's profit is given by:

$$\Pi_i = \wp_i \left[\pi_i(a) + \wp_j(a - c_0)q(\widehat{p}_j) - f\right] + \wp_i\wp_j \left[-a_i q(\widehat{p}_i) + a_j q(\widehat{p}_j)\right].$$

which is equal to

$$\Pi_i = \wp_i \left[\pi_i(a) + \wp_j(a - c_0)q(\widehat{p}_j) - f\right] - \kappa\wp_i\wp_j \left[\pi_i(a) - \pi_j(a)\right]. \tag{15}$$

Equation (15) shows well that our access pricing rules adds a sort of competition between the two firms in terms of the profit per customer $\pi_i(a)$ such that the firm which extracts more (less) surplus from consumers is punished (rewarded) in terms of the (additional) net access payment. The intensity of this competition increases with κ. Rearranging (15) gives

$$\Pi_i = \wp_i \left[(1 - \kappa\wp_j)\pi_i(a) - f\right] + \kappa\wp_i\wp_j \left[\pi_j(a) + (a - c_0)q(\widehat{p}_j)\right]. \tag{16}$$

As in the previous sections, we can apply a change of variables and let network i maximize profits by choosing p_i, $\widehat{p}_i$ and $\wp_i$, holding p_j, $\hat{p}_j$ and F_j fixed. Note that if p_i is changed while keeping $\wp_i$, $\hat{p}_i$, p_j, $\hat{p}_j$ and F_j fixed, then also $\wp_j$ will remain fixed. Note also that in a Logit model, we always have $\wp_i < 1$ and $\wp_j < 1$. Therefore, for $\kappa \leq 1$, maximizing Π_i with respect to on-net price p_i (keeping $\wp_i$ fixed) is equivalent to maximizing π_i. In other words, as long as $\kappa \leq 1$, κ does not affect the optimal choice of p_i. Since we know from the previous sections that network i chooses $p_i = c$ when $\kappa = 0$, network i chooses $p_i = c$ for $\kappa \leq 1$. For the same reason, network i chooses $\widehat{p}_i = c + a - c_0$ for $\kappa \leq 1$. Therefore, the class of access pricing rules induces networks to charge on-net price (off-net price) equal to on-net marginal cost (off-net marginal cost).

We now study the equilibrium fixed fee. From $p_i = c$ and $\widehat{p}_i = c + a - c_0$, we have $\pi_i(a) = F_i$. Then, (15) becomes

$$\Pi_i = \wp_i \left[F_i + \wp_j A(a) - f\right] - \kappa\wp_i\wp_j \left[F_i - F_j\right] \tag{17}$$

where $A(a) \equiv (a - c_0)q(c + a - c_0)$. Equation (17) clearly shows that our access pricing rule creates competition in terms of fixed fee: the firm charging a higher (lower) fixed fee is punished (rewarded) in terms of the (additional) net access payment. Rewriting equation (17) yields

$$\Pi_i = \wp_i \left[F_i - \wp_j \left[\kappa \left(F_i - F_j\right) - A(a)\right] - f\right].$$

The first order derivative with respect to F_i is given by:

$$\frac{\partial \wp_i}{\partial F_i}\left[F_i - \wp_j\left[\kappa\left(F_i - F_j\right) - A(a)\right] - f\right] + \wp_i\left[(1-\kappa\wp_j) - \left[\kappa\left(F_i - F_j\right) - A(a)\right]\frac{\partial \wp_j}{\partial F_i}\right].$$

Solving for a symmetric solution yields:

$$F = f - \wp A(a) - \frac{\wp}{\frac{\partial \wp_i}{\partial F_i}}\left[(1-\kappa\wp) + A(a)\frac{\partial \wp_j}{\partial F_i}\right]. \tag{18}$$

From $\frac{\partial \wp_i}{\partial F_i} < 0$, F decreases with κ when a is close to c_0. This is very intuitive since from (17), the extra competition in terms of fixed fee, generated by our access pricing rule, becomes more intensive as κ increases.

The equilibrium number of subscribers per firm is thus found by solving the system of equations (18) and (10).

Proposition 6 *Consider the retail benchmarking rules $\lambda(a,\kappa)$ defined by (14). Then, for $|a - c_0|$ small and any $\kappa \leq 1$,*

1. Each firm chooses on-net price equal to on-net marginal cost (c) and off-net price equal to off-net marginal cost ($c + a - c_0$).

2. The symmetric equilibrium is characterized by (18) and (10). In the equilibrium, the fixed fee strictly decreases with κ.

Corollary 1 *From a social welfare point of view, the retail benchmarking approach dominates the approach using a fixed reciprocal access charge for two reasons.*

1. For a given fixed access charge, it is possible to increase the number of subscribers by increasing κ from zero.

2. While in the case of fixed access charge, a distortion in off-net price is necessary to increase the number of subscribers, in the case of retail benchmarking, it is possible to increase the number of subscribers (by increasing κ from zero) while maintaining both on-net and off-net call princes equal to the marginal cost c.

Basically, our retail benchmarking rule creates an extra policy instrument that is absent in the fixed access charge approach. Namely, the social planner can increase the intensity of retail competition by increasing κ the intensity of the feedback from retail prices to access payment.

Furthermore, when there is a net network externality effect, it is also in the interest of firms to have the retail benchmarking approach with $a = c_0$ and with $\kappa > 0$. More precisely,

when $a = c_0$, as the equilibrium fixed fee decreases with κ, there is $\kappa^m > 0$ such that the equilibrium fixed fee for $\kappa = \kappa^m$ is exactly equal to the fixed fee that would be chosen by a monopolist owning both networks. On the other hand, if there is a net business stealing effect, firms would like to have an access charge $\tilde{a} < c_0$ in order to soften competition. However, there exists a retail benchmarking rule that allows the firms to make even higher profits but leave consumer surplus unaffected. Namely, the regulator can choose a retail benchmarking rule $\lambda(a', \kappa)$ with $\tilde{a} < a' < c_0$ and $\kappa < 0$ such that both rules lead to exactly the same market penetration and consumer surplus. Since the retail benchmarking rule leads to less distorted call volumes, it is more efficient, leads to higher total surplus and thus also to higher profits. Therefore,

Corollary 2 *(i) When $\mu < (1 - 2\wp^*)v$, the firms prefer the retail benchmarking approach (14) with $a = c_0$ and $\kappa = \min\{\kappa^m, 1\}$ to any fixed reciprocal access charge.*

(ii) When $\mu > (1 - 2\wp^*)v$, firms prefer some fixed reciprocal access charge $\tilde{a} < c_0$ in order to soften competition. However, the regulator can choose some rule $\lambda(a', \kappa)$ with $\tilde{a} < a' < c_0$ and $\kappa < 0$ such that both rules lead to exactly the same market penetration and consumer surplus. The retail benchmarking rule leads to less distorted call volumes and thus to higher profits.

When $a = c_0$, (18) becomes

$$F = f - \frac{\wp(1 - \kappa\wp)}{\frac{\partial \wp_i}{\partial F_i}}.$$

At symmetric equilibrium, we have

$$\frac{\partial \wp_i}{\partial F_i} = -\frac{\wp}{\mu} \frac{(1 - \wp)\mu - \wp(1 - 2\wp)v}{\mu - 2\wp(1 - 2\wp)v}.$$

Therefore, we have

$$F = f + \frac{\mu(1 - \kappa\wp)(\mu - 2\wp(1 - 2\wp)v)}{(1 - \wp)\mu - \wp(1 - 2\wp)v}. \tag{19}$$

Hence, if $F > f$ for $\kappa = 0$, then $F > f$ for $\kappa \leq 1$. On the other hand, (10) is given by

$$F = 2\wp v - W_0 - \mu \ln\left[\frac{\wp}{1 - 2\wp}\right]. \tag{20}$$

Note that (20) does not depend on κ. The equilibrium $(F, \wp)$ is determined by the two equations (19) and (20). Clearly, as κ increases, the equilibrium $(F, \wp)$ moves down following the curve of (20) and therefore, the fixed fee decreases while the number of subscribers increases.

4.3 Interpretation of the retail benchmarking rule

We now provide an economic interpretation of our access pricing rule. For this purpose, we consider $a = c_0$. Then, (14) is equivalent to

$$a_i - c_0 = \kappa(\wp_i + \wp_j)\left[s_i \frac{q_i^{on}}{q_i^{off}}(AP_i^{on} - c) + s_j(AP_i^{off} - c)\right] + \kappa(1 - \wp_i - \wp_j)\frac{F_i}{q_i^{off}} \tag{21}$$

where

$$s_i = \frac{\wp_i}{\wp_i + \wp_j}, AP_i^{on} = \frac{p_i q_i^{on} + F_i}{q_i^{on}}, AP_i^{off} = \frac{\widehat{p}_i q_i^{off} + F_i}{q_i^{off}}.$$

In other words, s_i is firm i's market share and AP_i^{on} (respectively, AP_i^{off}) is firm i's average on net price (off-net price).

To explain the rule (21), we consider some specific cases. First, without termination-based price discrimination and with full participation of consumers (i.e. $\wp_i + \wp_j = 1$), we are back to the rule (13) that we considered in Jeon and Hurkens (2008). Therefore, (21) generalizes (13) in two directions: termination-based price discrimination and partial participation.

Second, under full participation but with termination-based price discrimination, (21) becomes

$$a_i - c_0 = \kappa\left[s_i \frac{q_i^{on}}{q_i^{off}} AP_i^{on} + s_j AP_i^{off} - c\right]$$

In other words, our rule linearly links the access charge mark up to a weighted average retail price mark up in which the average price is a weighted sum of on-net average price and off-net average price and the weights depend on market shares (and are equal to market shares if $q_i^{on} = q_i^{off}$).

Third, under partial participation but without termination-based price discrimination, (21) becomes

$$a_i - c_0 = \kappa(\wp_i + \wp_j)\left[\frac{p_i q_i + F_i - c q_i}{q_i}\right] + \kappa(1 - \wp_i - \wp_j)\frac{F_i}{q_i}$$

Still our rule linearly links the access charge mark up to a weighted average retail price mark up in which the weights used are the fraction of subscribers and the fraction of non-subscribers. For the subscribers, the average retail price markup is computed as usual $(p_i q_i + F_i - c q_i)/q_i$; for the non-subscribers, the average retail price markup is given by putting the volume equal to zero in the numerator of the previous formula.

5 Retail Benchmarking Approach and First-Best

In this section, we assume that both the regulator and the firms have the same information (i.e. all of them know demand and cost structures) and show that there is a simple modification of our access pricing rule (14) that achieves the first-best outcome as a Nash equilibrium. Our aim is not so much to promote this modified access pricing rule but to illustrate the power of the retail benchmarking approach with respect to the approach based on a fixed access charge. Note that the first-best outcome is achieved when the firms charge the prices equal to the costs (i.e. $p_i = \widehat{p}_i = c$, $F_i = f$ for $i = 1, 2$) and this outcome can never be achieved under the approach based on a fixed access charge.

Let $\wp^{FB}$ be each network's number of subscribers in the first best. In a Logit model with duopoly, we have

$$0 < \wp^{FB} < 1/2.$$

Since the regulator knows demand and cost structure, the regulator knows $\wp^{FB}$. Define κ^* by $1 - \kappa^* \wp^{FB} = 0$; hence $\kappa^* > 2$. Let π_i denote network i's retail profit per customer gross of the fixed cost when $a = c_0$;

$$\pi_i \equiv \wp_i(p_i - c)q(p_i) + \wp_j(\widehat{p}_i - c)q(\widehat{p}_i) + F_i.$$

Then, for $a = c_0$, the access pricing rule (14) is given by

$$(a_i - c_0)\, q(\widehat{p}_i) = \kappa \pi_i \tag{22}$$

We modify it as follows:

$$(a_i - c_0)\, q(\widehat{p}_i) = \kappa^* \max\{\pi_i, f\} \tag{23}$$

In (23), we choose $\kappa = \kappa^*$ and add the max operator such that firm i cannot realize any further reduction of its access payment by pricing below costs. If i's access payment does not depend on its retail prices, firm i has no incentive to choose retail prices that give him a retail profit per customer below the fixed cost per customer. However, under our retail benchmarking approach, firm i may have an incentive to choose very low retail prices only to reduce its access payment such that its net access revenue more than covers its net retail loss. By adding the max operator, (23) makes such a deviation not profitable.

We now introduce one additional assumption:

Assumption 2 *An increase in F_i increases the number of subscribers to firm j.*

Assumption 2 is satisfied if μ is large enough. For instance, in a symmetric equilibrium with $p_i = \widehat{p}_i = c$, it holds if $\mu > (1 - 2\wp)v$ where $\wp$ is the number of subscribers to a firm in the symmetric equilibrium. In other words, the assumption holds if there is a net business-stealing effect.

Then, we have:

Proposition 7 *Suppose that the regulator proposes the access pricing rule (23). Then, under assumption 2, the first-best outcome can be implemented as a Nash Equilibrium: in the equilibrium, firm i chooses $p_i = \widehat{p}_i = c$, $F_i = f$ for $i = 1, 2$.*

Proof. Suppose that firm j uses $F_j = f, p_j = \widehat{p}_j = c$. Then, we distinguish two cases depending on whether $\wp_i > \wp^{FB}$ or $\wp_i < \wp^{FB}$.

Case 1: when $\wp_i > \wp^{FB}$. $\wp_i > \wp^{FB}$ implies that $\pi_i < f$. Then, firm i's profit is

$$\begin{aligned} \Pi_i &= \wp_i [\pi_i - f] - \wp_i \wp_j \kappa^* [f - f] \\ &= \wp_i [\pi_i - f] < 0. \end{aligned}$$

Case 2: when $\wp_i < \wp^{FB}$. $\wp_i < \wp^{FB}$ implies, from assumption 2, $1 < \wp_j \kappa^*$. Consider first $\pi_i \geq f$. Then, firm i's profit becomes

$$\Pi_i = \wp_i (1 - \wp_j \kappa^*) [\pi_i - f] \leq 0.$$

Consider now $\pi_i < f$. Then, firm i's profit is

$$\begin{aligned} \Pi_i &= \wp_i [\pi_i - f] - \wp_i \wp_j \kappa^* [f - f] \\ &= \wp_i [\pi_i - f] \leq 0. \end{aligned}$$

■

6 Conclusion

We studied how access pricing affects network competition when consumers' subscription demand is elastic and firms compete with non-linear tariffs and can use termination-based price discrimination. We first considered the standard approach based on a fixed and reciprocal

(per-minute) termination charge and found that both the firms and the regulator want to depart from cost-based termination charge (and hence want to distort call volumes) in order to affect consumer subscription. In particular, two opposing effects (softening competition and internalizing network externalities) are associated with a reduction in termination charge. The former decreases consumer surplus while the latter increases consumer surplus. The conflict and alignment of interests between the firms and the regulator in terms of preferred termination charge can be explained in view of the two effects.

It is useful to note that firms would like to choose termination charge in order to make the outcome of competition as close as possible to the outcome of a monopolist owning both networks. When termination charge is equal to termination cost, there are two possible cases: the consumer surplus under duopoly can be either larger or smaller than the one under monopoly. We called the first the case with a net business-stealing effect and the second the case with a net network externality effect. Obviously, in the first case, firms want to decrease consumer surplus, while in the second case, they want to increase consumer surplus. A surprising result is that in either case, firms prefer having termination charge below cost. The reason is that in the case of a net business-stealing effect, the softening-competition effect dominates the effect of internalizing network externalities and hence decreasing termination charge below cost is preferred by firms. In contrast, in the case of a net network externality case, the effect of internalizing network externalities dominates the softening-competition effect and hence decreasing termination charge below cost increases consumer surplus (and thereby expands consumer subscription). This suggests that in the first case, the regulator prefers termination charge above cost while in the second case, the regulator prefers termination charge below cost.

After studying the standard approach, we investigated the retail benchmarking approach. Since our previous paper (Jeon and Hurkens, 2008) considered the case without termination-based price discrimination and with inelastic subscription demand, this paper extended the previous approach to a more realistic setting. In addition, we find that for a given reciprocal fixed termination charge, we can find a family of access pricing rules parameterized such that all the access pricing rules in the family induce firms to charge on-net price equal to on-net cost and off-net price equal to off-net cost but the equilibrium fixed fee decreases with the strength of the feedback from retail prices to access payments. The result implies that the regulator can increase consumer subscription without distorting call volumes. Our access pricing rules intensify retail competition since a firm can reduce its access payment to rival firm(s) by reducing its retail prices. Furthermore, we show that when the regulator

and the firms have the same information about demand and cost structure, there is a simple modification of our rule that achieves the first-best outcome (i.e. firms charge prices just at costs and consumer subscription is maximized).

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