Dominance and Competitive Bundling*

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Abstract

We study how bundling affects competition between two asymmetric multi-product firms. One firm dominates the other in that it produces better products more efficiently. For low (high) levels of dominance, bundling intensifies (relaxes) price competition and lowers (raises) both firms' profits. For intermediate dominance levels, bundling increases the dominant firm's market share substantially, thereby raising its profit while reducing its rival's profit. Hence, the threat to bundle is then a credible foreclosure strategy. We also identify circumstances in which a firm that dominates only in some markets can profitably leverage its dominance to other markets by tying all its products.

Key words: Bundling, Tying, Leverage, Dominance, Entry Barrier

JEL Codes: D43, L13, L41.

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Does bundling or tying intensify competition? This is a classic question addressed in the literature on competitive bundling. While Matutes and Régibeau (1988) find that bundling intensifies competition in a symmetric duopoly, Kim and Choi (2015) and Zhou (2017) find that bundling can soften competition in a symmetric oligopoly when the number of firms is above a threshold which can be small. This paper contributes to a better understanding of how bundling affects price competition by considering an asymmetric duopoly in which one firm has symmetric dominance in all of its product markets.[1] We say that firm A is dominant in a product market if A's product gives a higher social surplus than its rival's product.[2] We obtain the following novel results: (i) for low levels of dominance, bundling reduces each firm's profit; (ii) for intermediate levels of dominance, bundling increases the dominant firm's profit but reduces the rival's profit; (iii) for high levels of dominance, bundling increases each firm's profit.

Our theoretical findings are relevant to antitrust policy. There are two widely publicized cases in which bundling dominant products was an issue. One is the GE/Honeywell merger case in which GE and Honeywell were dominant in the airplane engine and avionics market, respectively (Nalebuff, 2002). The European Commission (E.C.) opposed the merger because of the concern that the merged entity would drive out rivals by practicing bundling. The other is the Google-Android case: in 2016 the E.C. accused Google of abuse of dominance by forcing smartphone manufacturers using its Android operating system to pre-install Google Play and Google Search. The E.C. claimed that Google had more than 90% market share in each of the three respective markets (i.e., licensable smart mobile operating systems, app stores for the Android mobile operating system and general Internet search services) and expressed concerns about "the practices to close off ways for rival search engines to access the market" (European Commission, 2016).

We compare price competition under independent pricing with price competition under pure bundling in a multidimensional Hotelling setting. In each product market, the firms are located at the opposite ends of the unit interval; consumers' locations are distributed symmetrically around the center. The intuition for our results can be grasped by studying how bundling changes the relevant distribution of consumers and how dominance affects the

[1] Our model extends Matutes and Régibeau (1988) not only by allowing for dominance, but also by considering more general distributions of consumers' locations, and any number of products.

[2] The higher surplus can represent higher quality or lower production cost.

location of the marginal consumers. Together, these factors affect the number of marginal consumers and the allocation of firms' market shares, causing both demand elasticity and demand size effects.

What matters under bundling is the distribution of consumers' *average* locations. This distribution is (under general conditions) more-peaked: it is thicker at the center and thinner at the tails than the original distribution of locations.[3] The dominance level affects the location of the marginal consumers. As dominance increases, the location of the marginal consumers is closer to that of the dominated firm. In our baseline model, the dominant firm has the same dominance level in all product markets, so that the *average* dominance level, which matters under bundling, coincides with the dominance level for each product.

Without dominance, firms are symmetric and the marginal consumers are located at the center of the interval both under independent pricing and bundling. There are thus more marginal consumers under bundling because of the more-peakedness. This makes demand more elastic, intensifying price competition and lowering the profits of both firms. By contrast, when there is very strong dominance, the dominant firm has a market share close to one in either competition regime. Then the location of the marginal consumers is close to that of the dominated rival. Under bundling, the tail of the distribution is thinner and thus there are fewer marginal consumers, implying that demand is less elastic. Price competition is softened and the profits of both firms increase. For intermediate levels of dominance, there exists an asymmetric demand size effect which increases (reduces) the demand of the dominant firm (the dominated firm).[4] Suppose that the location of the marginal consumers under independent pricing is strictly between the center and the location of the dominated firm. To fix ideas, consider now bundling without changing price levels such that the average location of the marginal consumers under bundling is the same as the location of the marginal consumers under independent pricing. Then, because of the more-peakedness, bundling increases the demand of the dominant firm while it reduces that of the dominated firm. Even if both firms have incentives to change the prices, this asymmetric demand size effect determines the signs of profit changes for a range of intermediate levels of dominance such that bundling increases (reduces) the profit of the dominant firm (the

[3]This is similar to the well-known fact that the average of a random throw of two dice is more (less) likely to be between 3 and 4 (between 5 and 6) than the random throw of a single dice.

[4]This effect also exists for very small and very large dominance levels, but in these cases it is negligible.

dominated firm).

These results have an implication on the credibility of bundling when entry of firm B, the dominated firm, is endogenous. For intermediate levels of dominance, pure bundling of firm A is credible and builds an entry barrier. In contrast, for very high levels of dominance, bundling does not build a barrier to entry against B (but is still profitable for A). In this case, if A could dictate the terms of competition, the most effective way to deter entry would be to enforce competition in independent pricing, which is opposite to the insight of Whinston (1990).

We find that the welfare effect of bundling is non-monotonic in the dominance level. On the one hand, bundling reduces welfare by increasing mismatch between consumer preferences and products. On the other hand, bundling increases welfare if it increases the market share of firm A, because firm A is not aggressive enough from a welfare point of view, both under independent pricing and bundling. Bundling reduces welfare for low and high levels of dominance because in these cases the increase in market share of firm A is either positive but small or negative. However, for intermediate levels of dominance the demand size effect is strong and A's increase in market share may dominate the negative mismatch effect, in which case bundling improves welfare.

In Section 4, we depart from the baseline model by considering *asymmetric dominance* in the sense that firm A is dominant in some market(s) (called, tying good market(s)) but not dominant in other market(s) (called, tied good market(s)). We identify two conditions under which tying is profitable for firm A but hurts the rival: (i) price competition in the tied good market is sufficiently more fierce than in the tying good market; (ii) the tying firm leverages dominance from multiple markets. These results are relevant to the antitrust policy of tying. For instance, in the wholesale cable TV market, channel conglomerates may use bundling to foreclose competing channels. In fact, U.S. senator McCain introduced a bill in 2013 to encourage the wholesale and retail unbundling of programming.[5] Cablevision filed a lawsuit against Viacom as it considers that Viacom's obligation to acquire the bundle of core and suite networks forecloses Cablevision from distributing competing networks.

Finally, in Section 5 we consider competition between a generalist firm A (producing n different products) and n specialist firms (each producing exactly one of the products). When firm B is replaced by n specialists, bundling creates a Cournot complement problem: for any

[5] See Crawford and Yurukoglu (2012) for an empirical analysis of retail bundling in cable TV.

given price of A's bundle, the specialists choose prices higher than the price that would be chosen if they were integrated. This implies that A's profit under bundling is strictly higher when the rival firms are separate than when they are integrated. This suggests that bundling would have been credible for GE and Honeywell if the merger had been approved, as they faced specialist rivals.

1 Related literature

The bundling (or tying) literature can be divided into three categories. The papers in the first category study bundling as a price discrimination device for a monopolist (Schmalensee, 1984; McAfee, McMillan and Whinston, 1989; Salinger, 1995; Armstrong, 1996; Bakos and Brynjolfsson, 1999; Fang and Norman, 2006; Chen and Riordan, 2013; Menicucci, Hurkens and Jeon, 2015; Daskalakis, Deckelbaum and Tzamos, 2017). While the monopoly setting contrasts starkly with our duopoly model, it is worthwhile to mention that both Salinger (1995) (with uniform density) and Fang and Norman (2006) (with log-concave density) find the demand size effect that plays an important role in our results.

The second category is about competitive bundling where entry and exit is not an issue (Matutes and Régibeau, 1988; Economides, 1989; Carbajo, de Meza and Seidmann, 1990; Chen, 1997; Denicolo, 2000; Nalebuff, 2000; Gans and King, 2006; Armstrong and Vickers, 2010; Carlton, Gans and Waldman, 2010; Thanassoulis, 2011; Hahn and Kim, 2012; Kim and Choi, 2015; Zhou, 2017). Matutes and Régibeau (1988) show that bundling intensifies competition in the case of symmetric duopoly. Recently, Kim and Choi (2015) and Zhou (2017) consider symmetric oligopolies and show that the result of Matutes and Régibeau (1988) can be overturned when the number of firms is above some threshold $\hat{n} \geq 3$. For both papers, the results are a consequence of a demand elasticity effect which makes the demand under bundling less elastic, as it occurs in our model for large dominance. We contribute to the theory of competitive bundling by building a general framework that includes as a special case the model of Matutes and Régibeau (1988) and showing that the level of dominance of a firm is a crucial parameter such that their finding is completely reversed for strong dominance. Hahn and Kim (2012) also extend Matutes and Régibeau (1988) by introducing cost asymmetry, which generates results similar to our Proposition 3. However, we consider the general class of symmetric and log-concave densities, whereas they consider only the uniform distribution, and we provide a unified intuition based on the demand size and elasticity effects. Moreover, we identify novel conditions under which leverage of dominance into a dominated market is profitable.

The last category is about the leverage theory of bundling in which the main motive of bundling is to deter entry in the competitive segment of the market (Whinston, 1990; Choi and Stefanadis, 2001; Carlton and Waldman, 2002; Nalebuff, 2004; Peitz, 2008; Jeon and Menicucci, 2006, 2012). The existing theory of leverage considers leverage of monopoly power. For instance, Whinston (1990) finds that pre-commitment to tying builds an entry barrier by inducing the incumbent to be aggressive. However, tying is not credible as it also reduces the profit of the incumbent if entry occurs. Therefore, his logic works only for technical tying but not for contractual tying, which can be undone cheaply. We contribute to the leverage theory by studying leverage of dominance (and not of monopoly power) and its implications on entry barriers. We identify circumstances under which dominance in tying good market(s) can be credibly leveraged into a tied good market dominated by a rival, which provides a justification for the use of contractual bundling to deter entry.

2 Model

We consider competition between two firms A and B, each producing $n > 1$ different products. Let ij denote product j produced by firm i, for i =A, B and $j = 1, \ldots, n$. Each consumer has a unit demand for each product j.

We consider a model of both vertical and horizontal differentiation. Regarding the latter, let $s_j \in [0, 1]$ denote a generic consumer's location in terms of product j. For each product, firm A is located at $y_A = 0$ and firm B is located at $y_B = 1$ on a Hotelling segment. The gross utility that a consumer with location s_j obtains from consuming product ij is given by $v_{ij} - t_j|s_j - y_i|$, where $v_{ij} > 0$ is the same for all consumers, $t_j > 0$ is the usual (product specific) transportation cost parameter and $|s_j - y_i|$ denotes the distance between the consumer's and firm i's location. Utility is assumed to be additive over different products. We assume that v_{ij} is sufficiently high so that every consumer consumes one of the two competing products in market j, for any $j = 1, \ldots, n$. Hence, the products can be interpreted not only as products that can be independently consumed, but also as perfect complements.[6]

[6]In contrast with independent products, with perfect complements a consumer gets zero gross utility unless

For each firm i and product j, we assume that the marginal cost is large enough that each consumer buys at most one unit of product j. Without loss of generality, we simplify notation by normalizing all marginal costs to zero and interpret equilibrium prices as profit margins. A crucial role is played by the difference in surplus $\alpha_j = v_{Aj} - v_{Bj}$. We say that firm A is dominant in market j when $\alpha_j > 0$; α_j represents the level of A's dominance in market j.[7]

We assume that $s_1, ..., s_n$ are i.i.d., each with support $[0, 1]$, c.d.f. F and p.d.f. f such that $f(s) > 0$ for all $s \in (0, 1)$. Moreover, we assume that f is differentiable, symmetric around 1/2, that is $f(s) = f(1-s)$ for each $s \in [0, 1]$, and log-concave, that is $\log(f)$ is a concave function. This implies that f is weakly increasing on $[0, 1/2]$ and weakly decreasing on $[1/2, 1]$. It also implies that $\log(F)$ and $\log(1-F)$ are both concave, so that both $-F/f$ and $(1-F)/f$ are decreasing: see for example Bagnoli and Bergstrom (2005). For technical and expositional reasons we assume further that f is analytic on [0,1].[8]

Let p_{ij} be the price charged by firm i for product j under independent pricing. Under bundling, P_i denotes the price charged by firm i for the bundle of its products. We study the two following games of simultaneous pricing played by firms A and B:

- Game of independent pricing [**IP**]: firm A chooses p_{Aj} and firm B chooses p_{Bj} for all $j = 1, \ldots, n$.
- Game of pure bundling [**PB**]: firm A chooses P_A for its bundle of n products and firm B chooses P_B for its bundle of n products.

In section B.2 of the online appendix, we consider a third game in which $n = 2$, $\alpha_1 = \alpha_2$ and firms can use mixed bundling. We show that when firm A has a sufficiently large advantage, the equilibrium outcome of mixed bundling is the same as that of pure bundling.[9]

In Section 3, we study our baseline model of $\alpha_1 = ... = \alpha_n > 0$ and compare the two games. In addition, by making use of the comparison, we solve for the following three-stage

he buys a unit of all products. Our assumption on v_{ij} implies that, even with independent products and additive utilities, each consumer buys all products. Hence, there is here no difference between independent products and perfect complements.

[7] α_j would capture difference in marginal costs if they were different.

[8] f is analytic if it is infinitely differentiable and its Taylor series converges to f locally uniformly.

[9] Relatedly, in a monopoly context Menicucci, Hurkens and Jeon (2015) and Daskalakis, Deckelbaum and Tzamos (2017) provide conditions under which pure bundling is the best selling mechanism.

game of entry:[10]

- Stage one: firm B chooses between entering by incurring a fixed cost of entry $K > 0$ and not entering.
- Stage two: if B has entered, each firm chooses between IP and PB.
- Stage three: firms compete in the game determined by their choices at stage two.

According to the timing of the game, firm A cannot commit to bundling or independent pricing before entry can take place. This means in particular that a deterrence device which relies on commitment is not available under this timing. Notice that if in stage two at least one firm has chosen PB, then competition in stage three occurs between the two pure bundles.[11] Therefore competition in independent prices occurs if and only if both firms have chosen IP. Also, there always exists an equilibrium in which both firms choose PB at stage two, but it may involve playing a weakly dominated strategy. We impose that firms do not play weakly dominated strategies, therefore (IP, IP) is the outcome only if this is the preferred outcome for both firms.

3 Competition between generalists: symmetric markets

In this section, we consider the baseline model of competition between firm A and firm B where all n markets are symmetric: firm A has a symmetric dominance (i.e., $\alpha = \alpha_1 = ... = \alpha_n > 0$) and the transportation cost is symmetric (i.e., $t = t_1 = ... = t_n > 0$). After studying the game of independent pricing and that of pure bundling, we compare the two. We consider asymmetric dominance levels and transportation costs in Section 4.

3.1 Independent Pricing

When firms compete in independent prices, we can consider each market in isolation. Moreover, since markets are symmetric, we suppress the index to product j and restrict attention

[10] Peitz (2008) studies the same game in the context of leveraging monopoly power.

[11] Indeed, suppose that firm A, for instance, has chosen PB and firm B has chosen IP. Then each consumer either buys the pure bundle of A or the n products of firm B, which are therefore viewed as a bundle.

to price competition on the Hotelling segment in one market. Given prices p_A and p_B, the indifferent consumer is located at

$$x(\alpha, p_A, p_B) = \frac{1}{2} + \sigma\alpha - \sigma(p_A - p_B), \tag{1}$$

where

$$\sigma = 1/(2t).$$

For simplicity, we will often suppress the arguments and simply write x for the location of the indifferent consumer.

We suppose for now that the distribution and the parameters are such that independent pricing leads to an interior equilibrium, i.e., both firms obtain a positive market share.[12] Then the first-order conditions must be satisfied at the equilibrium prices. Since marginal costs are assumed to be zero, the profit functions are

$$\pi_A = p_A F(x), \qquad \pi_B = p_B(1 - F(x)),$$

and the first-order conditions are[13]

$$0 = F(x) - \sigma p_A f(x), \qquad 0 = 1 - F(x) - \sigma p_B f(x).$$

If p_A^* and p_B^* are the equilibrium prices and x^* denotes the equilibrium location of the indifferent consumer, then we have

$$x^* = x(\alpha, p_A^*, p_B^*) = \frac{1}{2} + \sigma\alpha - \sigma(p_A^* - p_B^*) = \frac{1}{2} + \sigma\alpha + \frac{1 - 2F(x^*)}{f(x^*)}.$$

Hence, the equilibrium location of the indifferent consumer is a fixed point of the mapping:

$$X^\alpha : x \mapsto \frac{1}{2} + \sigma\alpha + \frac{1 - 2F(x)}{f(x)}. \tag{2}$$

Notice that $\frac{1-2F(x)}{f(x)} = -\frac{F(x)}{f(x)} + \frac{1-F(x)}{f(x)}$. As we mentioned in Section 2, log-concavity of f implies that both $-\frac{F(x)}{f(x)}$ and $\frac{1-F(x)}{f(x)}$ are decreasing. Hence, X^α is weakly decreasing. This, jointly with $X^\alpha(1/2) > 1/2$, implies that a unique fixed point $x^* < 1$ exists provided that

[12] Proposition 1 characterizes when an interior equilibrium exists.

[13] Notice that $\frac{d\pi_A}{dp_A} = 0$ suffices to maximize π_A because $\frac{d\pi_A}{dp_A} = F(x)[1 - \sigma p_A \frac{f(x)}{F(x)}]$, and because log-concavity of F implies that $\frac{f(x)}{F(x)}$ is decreasing in x, and thus increasing in p_A. Hence, if p_A^* solves the first-order condition, then $\frac{d\pi_A}{dp_A} < 0$ for $p_A > p_A^*$ and $\frac{d\pi_A}{dp_A} > 0$ for $p_A < p_A^*$. A similar argument reveals that $\frac{d\pi_B}{dp_B} = 0$ suffices to maximize π_B with respect to p_B.

$\lim_{x\uparrow 1} X^\alpha(x) < 1$. The equilibrium prices are then also unique. Clearly, at $\alpha = 0$ we have $x^* = 1/2$ and Proposition 1(i) establishes that x^* is increasing in α, hence $x^* > 1/2$ for $\alpha > 0$. If α is sufficiently large and $f(1) > 0$, then $x^* = 1$.

Proposition 1 (Independent Pricing)**.** *(i) Suppose that $(\sigma\alpha - 1/2)f(1) < 1$. Then the independent pricing game has a unique and interior equilibrium, characterized by the unique fixed point $x^*(\alpha)$ of X^α, and $x^*(\alpha) \in [\frac{1}{2}, 1)$. The function $x^*(\alpha)$ is increasing and concave for $\alpha \geq 0$. The equilibrium prices (in each market) are*

$$p_A^*(\alpha) = \frac{F(x^*(\alpha))}{\sigma f(x^*(\alpha))}, \qquad p_B^*(\alpha) = \frac{1 - F(x^*(\alpha))}{\sigma f(x^*(\alpha))}.$$

The equilibrium profits (in each market) are

$$\pi_A^*(\alpha) = \frac{F(x^*(\alpha))^2}{\sigma f(x^*(\alpha))}, \qquad \pi_B^*(\alpha) = \frac{(1 - F(x^*(\alpha)))^2}{\sigma f(x^*(\alpha))}.$$

p_A^* and π_A^* are increasing in α, while p_B^* and π_B^* are decreasing in α.

(ii) Suppose that $(\sigma\alpha - 1/2)f(1) \geq 1$. Then the independent pricing game has a unique equilibrium, and it is such that firm A's market share is 1. The equilibrium prices and profits (in each market) are

$$p_A^*(\alpha) = \pi_A^*(\alpha) = \alpha - 1/(2\sigma), \qquad p_B^*(\alpha) = \pi_B^*(\alpha) = 0.$$

Note that for any prices p_A and p_B, the demand for firm A is given by $D_A(p_A, p_B) = F(x(\alpha, p_A, p_B))$. So, the elasticity of D_A with respect to p_A equals

$$\varepsilon_A(p_A, p_B) = \frac{\sigma f(x) p_A}{F(x)}. \tag{3}$$

Similarly, the demand for firm B is $1 - F(x(\alpha, p_A, p_B))$ and its elasticity with respect to p_B equals

$$\varepsilon_B(p_A, p_B) = \frac{\sigma f(x) p_B}{1 - F(x)}. \tag{4}$$

In particular, at the equilibrium prices p_A^* and p_B^* the elasticity of demand with respect to one firm's own price is equal to one for each firm: $\varepsilon_A(p_A^*, p_B^*) = 1 = \varepsilon_B(p_A^*, p_B^*)$. This resembles the well-known inverse elasticity rule for a monopolist with zero marginal cost. This is so because, for a given price of the rival, each firm acts as a monopolist with respect to its own demand. Hence, each firm's reaction function is determined by this inverse elasticity rule.

3.2 Pure Bundling

The analysis performed for independent pricing straightforwardly extends to bundling since competition under bundling can be considered as a competition between two firms, each offering one product – in fact, a bundle. The only difference is that we should use the density f_n of the average location $(s_1 + \ldots + s_n)/n$, and not the density f of the individual location. Note that the density function of a sum of n i.i.d. random variables with p.d.f. f is obtained by the n-fold convolution of f. The density function of the average, f_n, is then obtained by rescaling. It follows that f_n has two properties that are relevant for our analysis of pure bundling. First, f_n is log-concave and symmetric because f is (see An, 1998, Cor. 1). Second, $f_n(1) = 0$.

Given P_A and P_B chosen by the firms, let $p_A = P_A/n$ and $p_B = P_B/n$ denote the prices per product. Let x_n denote the average location of the indifferent consumer, which is again given by:

$$x_n = x(\alpha, p_A, p_B) = \frac{1}{2} + \sigma\alpha - \sigma\left(p_A - p_B\right).$$

The equilibrium bundle prices are found in a way very similar to the analysis of independent pricing. Let us thus define

$$X_n^\alpha : x \mapsto \frac{1}{2} + \sigma\alpha + \frac{1 - 2F_n(x)}{f_n(x)}. \tag{5}$$

Since f_n is log-concave, we obtain (as above) that X_n^α is decreasing in x. Since $f_n(1) = 0$, X_n^α always admits a unique fixed point $x_n^*(\alpha) < 1$, and equilibrium prices and profits can be expressed in terms of this fixed point. Hence, under pure bundling we always obtain a unique and interior equilibrium in which both firms have a positive market share.

Proposition 2 (Pure Bundling)**.** *The pure bundling pricing game has a unique equilibrium, characterized by the unique fixed point $x_n^*(\alpha)$ of X_n^α, and $x_n^*(\alpha) \in [\frac{1}{2}, 1)$. The function $x_n^*(\alpha)$ is increasing and concave for $\alpha \geq 0$. The equilibrium bundle prices are*

$$P_{n,A}^*(\alpha) = \frac{nF_n(x_n^*(\alpha))}{\sigma f_n(x_n^*(\alpha))}, \qquad P_{n,B}^*(\alpha) = \frac{n(1 - F_n(x_n^*(\alpha)))}{\sigma f_n(x_n^*(\alpha))}.$$

The total equilibrium profits are

$$\Pi_{n,A}^*(\alpha) = \frac{nF_n(x_n^*(\alpha))^2}{\sigma f_n(x_n^*(\alpha))}, \qquad \Pi_{n,B}^*(\alpha) = \frac{n(1 - F_n(x_n^*(\alpha)))^2}{\sigma f_n(x_n^*(\alpha))}.$$

$P_{n,A}^*$ and $\Pi_{n,A}^*$ are increasing in α while $P_{n,B}^*$ and $\Pi_{n,B}^*$ are decreasing in α.

Note that the elasticity of demand for A's bundle (w.r.t. own price per product) is given by

$$\tilde{\varepsilon}_A(p_A, p_B) = \frac{\sigma f_n(x) p_A}{F_n(x)}. \tag{6}$$

As under independent pricing, at the equilibrium bundle prices the elasticity of demand with respect to a firm's own price is equal to one.

3.3 Independent pricing vs. pure bundling

In this section we study how bundling affects each firm's profit in comparison with independent pricing. We first do this for the special case where each firm produces just two products and where consumers' locations for each product are uniformly distributed. This case allows explicit expressions and is therefore straightforward. It shows that both firms lose (gain) from bundling for low (high) levels of dominance, and that only the dominant firm benefits from bundling for intermediate levels. We then show that this result extends to any number of products and any symmetric log-concave density function. Since the proof for the general case is quite technical, we leave that to the Appendix and here provide only the intuition, based on the demand size and demand elasticity effects of bundling. We will explain these effects using a few important properties of the distribution of the average location.

3.3.1 Uniform distribution and two products

In the special case of the uniform distribution, we have $f(x) = 1$, $F(x) = x$ and, for $x \geq 1/2$, $f_2(x) = 4(1-x)$, $F_2(x) = 1 - 2(1-x)^2$. These functions are depicted in Figure 1. Fixing $t = 1$ (or $\sigma = 1/2$), we can get explicit expressions for equilibrium prices and profits under independent pricing and bundling when $n = 2$ by substituting $x^* = (3+\alpha)/6$ and $x_2^* = \left(7 + \alpha - \sqrt{9 - 2\alpha + \alpha^2}\right)/8$ into the expressions of Propositions 1 and 2.[14]

It is easily verified (numerically) that firm A benefits from bundling if and only if $\alpha > 1.415$ and that firm B benefits from bundling if and only if $\alpha > 2.376$. Defining dominance regions

$$\mathrm{DL}^{--} = [0, 1.415), \quad \mathrm{DL}^{+-} = (1.415, 2.376), \quad \mathrm{DL}^{++} = (2.376, \infty),$$

[14] The condition $(\sigma\alpha - 1/2)f(1) \geq 1$ in Proposition 1(ii) is equivalent to $\alpha \geq 3$, hence $x^* = (3+\alpha)/6$ if $\alpha < 3$, $x^* = 1$ if $\alpha \geq 3$.

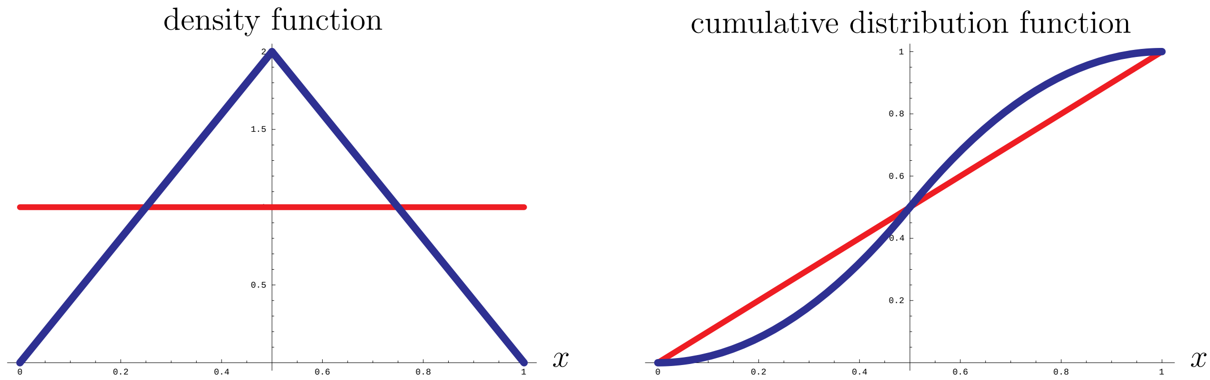

Figure 1: Density and distribution functions for the uniform case. The thicker (blue) graphs correspond to average valuations (bundling).

we see that both firms lose from bundling when $\alpha \in \mathrm{DL}^{--}$, firm A gains while firm B loses from bundling when $\alpha \in \mathrm{DL}^{+-}$, and both firms gain from bundling when $\alpha \in \mathrm{DL}^{++}$.

3.3.2 The general case

We now consider the general case with n products and any symmetric log-concave f. We will establish the existence of three dominance level regions DL^{--}, DL^{+-} and DL^{++} with the respective effects on the profits of firms A and B under bundling. Unfortunately, direct comparison of the profits is hard because not only are the relevant distribution functions (f and f_n) different under independent pricing and bundling, but so are the equilibrium locations of the indifferent consumers ($x^*(\alpha)$ and $x_n^*(\alpha)$). One exception is the symmetric case without dominance (i.e., when $\alpha = 0$) because then clearly $x^* = 1/2 = x_n^*$ and $F(1/2) = 1/2 = F_n(1/2)$. Direct comparison of the expressions for profits in Propositions 1 and 2 shows that both firms lose from bundling if and only if $f_n(1/2) > f(1/2)$. We show this to be true in Lemma 1 below. Hence, this already generalizes the findings of Matutes and Régibeau (1988) that bundling lowers profits in a symmetric duopoly.

In order to establish our results in the case of dominance, we need some further important properties of the density f_n of the average location. First, f_n is strictly more-peaked around 1/2 (the mean) than the density f of the individual location (Proschan, 1965).[15] [16] That is,

[15]In fact, for all $t_1, \ldots, t_n > 0$, the weighted average $(\sum_{j=1}^n t_j s_j)/\sum_{j=1}^n t_j$ is distributed with a log-concave density function that is more-peaked around the mean than that of the original variable.

[16]Observe that for density functions that are not log-concave, the average is not necessarily more-peaked than the original distribution (see for instance the Cauchy distribution). This explains our restriction to

for any $z \in (0, 1/2)$,

$$\int_z^{1-z} f(s)ds < \int_z^{1-z} f_n(s)ds.$$

The more-peakedness of f_n can be equivalently expressed as $F_n(x) > F(x)$ for all $x \in (1/2, 1)$. In fact, we have $F_k(x) > F_{k-1}(x)$ for all $x \in (1/2, 1)$, for $k = 2, ..., n$. Second, Lemma 1 states two useful properties of the ratio between the density function of the average and that of the individual locations.[17]

Lemma 1. *Let f be a log-concave density function which is symmetric around $1/2$ with support $[0, 1]$. Let f_n be the density function of the average of $n > 1$ random variables that are i.i.d. according to the density function f. Then we have:*

(i) $f_n(1/2) > f(1/2)$;

(ii) $\lim_{s \uparrow 1} \frac{f_n(s)}{f(s)} = 0$.

In sum, the distribution of the average is less dispersed than the distribution of the original random variable. There is relatively strictly more weight at the mean and much less weight at the extremes. Figure 1 illustrates the more-peakedness and the properties of Lemma 1 for the case of the uniform distribution with $n = 2$.

Using these general properties, we now proceed to explain how bundling causes demand size and demand elasticity effects. Jointly, they determine how firms' profits are affected by bundling. Rather than comparing the equilibrium profits under independent pricing and bundling directly, we use a two-step procedure by considering an intermediate step where firms sell the products in a bundle but charge prices corresponding to the equilibrium under independent pricing.

Demand size effect. Suppose that initially firms A and B sell their products independently and set the equilibrium prices p_A^* and p_B^* for each product. At these prices, firm A has demand $F(x^*)$. Suppose now that each firm bundles its n products, without changing the price per product: $P_A = np_A^*$ and $P_B = np_B^*$. Then the indifferent consumers are those whose *average* location is equal to x^*. The demand for A's bundle is thus equal to $F_n(x^*)$ and the demand for B's bundle is $1 - F_n(x^*)$. Since F_n is more-peaked around the mean, $F_n(x^*) > F(x^*)$

log-concave densities.

[17]We develop Lemma 1 because the result does not follow generally from Proschan (1965). We use here the fact that the support of the distributions is assumed compact. For example, property (i) of Lemma 1 does not hold for the Laplace distribution defined on the real line.

whenever $1/2 < x^* < 1$. Hence, bundling — without changing prices per product — leads to a higher demand for the dominant firm. We call the difference $F_n(x^*) - F(x^*)$ the demand size effect of bundling. Note that this difference is tiny when $x^* \approx 1/2$ or $x^* \approx 1$, and that it is maximal at $x^* = \hat{x}$, where $\hat{x} \in (\frac{1}{2}, 1)$ is defined by $f(\hat{x}) = f_n(\hat{x})$.[18]

Demand elasticity effect. After bundling, the firms will have incentives to change their prices. From Sections 3.1 and 3.2, we know that each firm chooses its price optimally by making the elasticity of own demand equal to one. In particular, this is why $p_A^* = F(x^*)/(\sigma f(x^*))$. After bundling, but before changing prices, the elasticity of demand for A's bundle is given by (6) and therefore equal to

$$\tilde{\varepsilon}_A = \sigma \frac{f_n(x^*)}{F_n(x^*)} p_A^* = \frac{f_n(x^*)}{f(x^*)} \frac{F(x^*)}{F_n(x^*)}$$

and this will in general not equal one. This change in the elasticity of demand after bundling is what we call the demand elasticity effect. Clearly, for $x^* > \hat{x}$ we have $f_n(x^*) < f(x^*)$ and then from $F_n(x^*) \geq F(x^*)$ we obtain $\tilde{\varepsilon}_A < 1$. In this case, bundling makes demand less elastic and the dominant firm wants to raise its price. In particular, for large dominance, x^* is close to 1 and the demand elasticity effect is very large because of Lemma 1(ii). On the contrary, for low dominance, $x^* \approx 1/2$, $F(x^*) \approx F_n(x^*)$ but then $f_2(x^*) > f(x^*)$ by Lemma 1(i). Therefore, we have $\tilde{\varepsilon}_A > 1$. In this case, bundling makes demand more elastic and the dominant firm wants to lower its price.

For very low levels of dominance such that $x^* \approx 1/2$, both firms are similar and hence bundling makes demand more elastic for both firms. Therefore, both firms want to lower their price after bundling and thus lose from bundling. This result also follows directly from the fact that profits are continuous in the dominance level and that both firms lose from bundling when there is no dominance, as we established before.

For a range of intermediate levels of dominance, bundling affects the profits of the two firms in different ways because of the demand size effect. Even if both firms have incentives to change their price after bundling, this effect is dominated by the demand size effect when the latter is relatively large, which implies that bundling then increases the profit of the dominant firm and reduces the profit of the dominated firm.

[18] The existence of such $\hat{x}$ is guaranteed by Lemma 1. For the sake of exposition, we assume here it is unique.

Finally, for very high levels of dominance, the dominant firm's demand becomes very inelastic after bundling, giving incentives to raise its price substantially. This shifts the location of the indifferent consumer away from the dominated firm, raising the demand of firm B. Since for high dominance levels the demand size effect is negligible, the overall effect is that firm B benefits from bundling. We can make this statement easily precise if we suppose that $f(1) > 0$.[19] Then Proposition 1(ii) reveals that under independent pricing, B has zero market share, zero profit, and sets $p_B^* = 0$. Bundling cannot reduce the profit of firm A because by charging $P_A = np_A^*$, it still sells all products to all consumers (since necessarily $P_B \geq 0$) and earns the same profit as under independent pricing. But in fact, firm A can do even better by raising its price a bit as its demand is very inelastic. Recall that the profit of firm A is $P_A F_n(x)$, and a marginal increase of its price, ΔP_A, raises profits by $\Delta P_A(F_n(x) - \sigma \frac{P_A}{n} f_n(x))$, which is strictly positive at $x = 1$ because $f_n(1) = 0$.[20] Moreover, $P_A > np_A^*$ benefits firm B as it allows B to charge a strictly positive price P_B and nevertheless have a positive market share, yielding a strictly positive profit. Hence, bundling makes both firms better off when α is large. Lemma 2 below shows more generally that both firms prefer bundling when bundling increases firm B's market share, and that there exist dominance levels for which only firm A benefits from bundling.

Lemma 2. *We have the following implications for each dominance level α:*
(i) If α is such that bundling strictly raises firm B's market share, then bundling raises firm B's profit.
(ii) If α is such that bundling raises firm B's profit, then bundling raises firm A's profit.
(iii) Neither the reverse of (i) nor the reverse of (ii) holds.

An immediate consequence of Lemma 2 is that there are three regions of dominance levels, DL^{--}, DL^{+-} and DL^{++}, with distinct effects of bundling on the firms' profits. For one set of dominance levels, DL^{--} (including $\alpha = 0$), both firms are hurt by bundling; for another set, DL^{+-}, bundling hurts only firm B; for a final set, DL^{++} (including high values of α), both firms benefit from bundling. It never happens that bundling hurts the dominant firm while benefiting the rival. We conjecture that these dominance sets are convex sets

[19] Our formal proof does allow for $f(1) = 0$. It does not change our conclusions but complicates the argument.

[20] We find at work here the same principle which makes it optimal not to sell to all consumers for a multi-product monopolist: see Armstrong (1996).

(i.e., intervals), but have been unable to prove it for general log-concave symmetric density functions and number of products.[21] We therefore summarize our central result as follows.

Proposition 3 (Independent pricing vs. bundling)**.**

(i) There exist threshold levels $0 < \underline{\alpha} \le \alpha_{\pi A} < \alpha_{\pi B} \le \bar{\alpha}$ *such that bundling strictly benefits firm A and hurts firm B when* $\alpha \in (\alpha_{\pi A}, \alpha_{\pi B})$*, strictly hurts both firms when* $\alpha \in [0, \underline{\alpha})$ *and strictly benefits both firms when* $\alpha > \bar{\alpha}$*.*

(ii) If the dominance level sets DL^{--}*,* DL^{+-} *and* DL^{++} *are convex, then* $\underline{\alpha} = \alpha_{\pi A}$ *and* $\alpha_{\pi B} = \bar{\alpha}$ *so that (a) profits of firm A are strictly higher under bundling if and only if* $\alpha > \alpha_{\pi A}$ *and (b) profits of firm B are strictly higher under bundling if and only if* $\alpha > \alpha_{\pi B}$*.*

For the case of uniform distributions, we can numerically determine the different dominance level sets for different numbers of products. Let $\underline{\alpha}_n$ (respectively, $\bar{\alpha}_n$) denote the cutoff value for which firm A (respectively B) is indifferent between independent pricing and pure bundling when there are n products. Then $\underline{\alpha}_2 = 1.41$, $\underline{\alpha}_3 = 1.42$, $\underline{\alpha}_4 = 1.39$, $\underline{\alpha}_8 = 1.29$, $\underline{\alpha}_{16} = 1.19$ and $\bar{\alpha}_2 = 2.38$, $\bar{\alpha}_3 = 2.54$, $\bar{\alpha}_4 = 2.64$, $\bar{\alpha}_8 = 2.77$, $\bar{\alpha}_{16} = 2.85$. Bundling more and more products makes wider the dominance region DL^{+-}, where only firm A benefits from bundling, which plays a role for the result described in Proposition 6.[22]

Our results can be extended to the case of positive correlation in tastes. Suppose that a fraction $\rho \in (0, 1]$ of consumers have perfectly correlated locations, while the rest have locations which are i.i.d. as above. Given (ρ, ρ') satisfying $1 \ge \rho > \rho' > 0$, the distribution of the average location for ρ is less peaked than that of the average location for ρ'. Therefore, a greater positive correlation weakens both the demand size effect and the demand elasticity effect.[23]

3.4 Entry deterrence

We are now in a position to solve the three-stage game outlined in Section 2 by backward induction. In the last stage firms choose the equilibrium prices corresponding to the bundling game whenever at least one firm has chosen PB. This is so because if, for example, firm

[21] We can numerically show this to be true for the uniform distribution and $2 \le n \le 32$ and for symmetric beta-distributions $f(s) = (s(1-s))^{\beta}$ for integers $1 \le \beta \le 10$ and $n = 2$.

[22] However, note that $\underline{\alpha}_n$ is not monotonic at $n = 3$ as we have $\underline{\alpha}_3 > \underline{\alpha}_2 > \underline{\alpha}_4$.

[23] In the case of the uniform distribution with $n = 2$, we find that the two threshold values of α_2 decrease with $\rho \in (0, 1)$. However, $\underline{\alpha}_2$ decreases more slowly than $\bar{\alpha}_2$.

A chooses PB and firm B chooses IP, then effectively competition will be in bundles, and equilibrium prices will be $P^*_{n,A}$ for A's bundle, and $P^*_{n,B}/n$ for each of B's individual products. Substituting equilibrium payoffs from Stage 3 yields the following game to be played in Stage 2, where K denotes the fixed cost of entry of firm B.

	PB	IP
PB	$\Pi^*_{n,A}, \Pi^*_{n,B} - K$	$\Pi^*_{n,A}, \Pi^*_{n,B} - K$
IP	$\Pi^*_{n,A}, \Pi^*_{n,B} - K$	$n\pi^*_A, n\pi^*_B - K$

Of course, in this reduced form game, both firms have a weakly dominant strategy. It is a weakly dominant strategy for firm A to choose PB when $\alpha \in \mathrm{DL}^{+-} \cup \mathrm{DL}^{++}$ and to choose IP otherwise. Similarly, it is a weakly dominant strategy for firm B to choose PB when $\alpha \in DL^{++}$ and to choose IP otherwise.

The bundling outcome will prevail whenever firm A benefits from bundling and it will then be enforced by firm A. For very low levels of dominance, firm A may be tempted to threaten to use bundling in order to deter entry, as this would lower firm B's profit. However, such a threat is not credible because firm A would choose independent pricing once B has entered. For (very high) levels of dominance for which bundling is profitable for firm B, firm A cannot use bundling to deter entry. In fact, in order to deter entry, firm A would need to threaten to use IP. Not only is this not credible, but firm B can in fact force bundling by choosing PB unilaterally. Only for intermediate levels of dominance, bundling is profitable for A (and thus credible) and hurts firm B. Hence, bundling can then be used as a foreclosure strategy if it reduces B's profit below B's entry cost.[24]

3.5 Social welfare

In this subsection we study and compare static social welfare (defined as the sum of producers' and consumers' surplus) under independent pricing and bundling. We focus on the case

[24] In online Appendix B.2, we consider $n = 2$ and show that when firm A has a sufficiently large advantage, the equilibrium outcome of mixed bundling is the same as that of pure bundling. Hence, if we replace Stages 2 and 3 with a single stage in which both firms can choose any tariff in mixed bundling, the outcome of the sequential game remains unchanged as long as A's dominance is large enough. In the case of the uniform distribution, mixed bundling leads to the pure bundling outcome for $\alpha \geq 9t/8$.

of $f(1) > 0$. We find that the effects of bundling are non-monotonic in the dominance level. Recall from Propositions 1 and 2 that the market share is always interior under bundling while, under independent pricing, firm A's share becomes one when $(\sigma\alpha - 1/2)f(1) \geq 1$. We have:

Proposition 4 (Welfare)**.** *(i) Both under independent pricing and under bundling, the market share of firm A is too low from the point of view of social welfare for any* $\alpha > 0$ *as long as it is interior.*

(ii) Suppose that $f(1) > 0$. *Then bundling reduces social welfare both for* $\alpha(\geq 0)$ *small enough and for* α (≥ 0) *large enough (in particular, when* $(\sigma\alpha - 1/2)f(1) \geq 1$*). For intermediate values of* α*, bundling may increase or reduce social welfare. When* F *is the uniform distribution and* $n = 2$*, bundling increases social welfare if* α *is between* $1.071t$ *and* $2.306t$ *and reduces social welfare otherwise.*

The dominance of firm A over firm B implies that the socially optimal location of the indifferent consumer is greater than $\frac{1}{2}$, but it turns out that in equilibrium firm A is often not aggressive enough, and the location of the indifferent consumer is closer to $\frac{1}{2}$ than the socially optimal location. There is an exception for the case of competition under IP when $(\sigma\alpha - 1/2)f(1) \geq 1$, because then it is socially optimal that each consumer buys from firm A, and $x^*(\alpha) = 1$ holds because of Proposition 1(ii). This implies that IP generates a higher social welfare than PB when α is large, since IP induces then the socially optimal outcome that all consumers buy from firm A. Conversely, when α is close to zero the firms are almost symmetric, thus they have about the same market share and the difference in social welfare between IP and PB is mainly determined by transportation costs. The latter are smaller under IP than under PB because PB prevents consumers from freely selecting their preferred combinations of products (that is, they cannot mix and match). Hence PB generates a lower social welfare.

However, when α takes on intermediate values PB may significantly increase the market share of firm A with respect to IP. This brings the economy closer to the socially optimal allocation, and this effect may outweigh the effect of transportation costs, such that PB increases social welfare. For the uniform distribution, part (ii) of the proposition identifies the set of values of α for which this occurs.

Our welfare comparison above has been static in the sense that we have considered a given duopolistic market structure. However, we know from Subsection 3.4 that bundling may help firm A to erect an entry barrier against firm B. For instance, for the uniform distribution and $n = 2$, bundling is credible and reduces B's profit for $\alpha \in (1.415t, 2.376t)$, which largely overlaps with the interval for which bundling increases static welfare. Therefore, one should be very cautious in generating policy implications on bundling from static welfare analysis.

4 Credible leverage of dominance

In the previous section, we considered only one type of asymmetry, captured by the dominance level α of firm A, and assumed that all markets were the same. In order to relate our results to the leverage theory, we now consider asymmetric markets. In particular, we assume that A is dominant in some market (called, tying product market) but faces an equally strong competitor in some other market(s) (called, tied product market).[25] We investigate whether bundling can be profitable, that is, whether leverage is feasible.

With symmetric markets, the effect on profits of bundling could be decomposed into a demand size and a demand elasticity effect. However, the demand size effect does not straightforwardly extend to asymmetric markets because bundling at equal total price reduces demand for the tying product and increases demand for the tied product. When total demand (in terms of total units sold) stays the same, bundling reduces profits when the profit margin in the tying product is higher than in the tied product, which is typically the case. In particular, for the profit of firm A to increase it is necessary (but not sufficient) that firm A enjoys a positive "total" demand size effect.

It is not obvious that a firm that is dominant in one market can profitably leverage this dominance to a second market by bundling the two products. In fact, in the case of uniform distributions of consumers' locations and equal transportation cost, we can show that bundling is not credible as it reduces A's profit for any $\alpha_1 \in [0, 3t]$ and $\alpha_2 = 0$.

Nevertheless, we identify two circumstances in which A can leverage dominance and increase overall profits. First, we show that (for some parameters) leverage of dominance is possible and profitable when the firm is sufficiently dominant in a market where price competition is relatively weak compared to that of the tied product market. Second, we show that if the firms compete in $n \geq 3$ markets and one firm is dominant in sufficiently

[25] By continuity, our results can be extended to situations where A is slightly dominated in the tied product market.

many of those, then bundling is again profitable. These results are relevant to the antitrust policy of tying as antitrust authorities are often concerned about situations in which the tied product is inferior to or as good as the competing product.

4.1 Asymmetric intensity of competition

In this subsection, we assume that firm A is dominant in market 1 but not in market 2 (*i.e.*, $\alpha_1 > 0 = \alpha_2$), but price competition is more intense in market 2 than in market 1. We capture this difference in competition intensity in two complementary ways. First, we assume that, while in market 1 the location of consumers is uniformly distributed, in market 2 it is distributed according to a symmetric (and log-concave) beta distribution such that $f^\beta(x) = x^{\beta-1}(1-x)^{\beta-1}/B(\beta,\beta)$, where $B(\beta,\beta) = \int_0^1 s^{\beta-1}(1-s)^{\beta-1}ds$ and $\beta \geq 1$. Notice that f^1 coincides with the uniform density, but for $\beta > 1$ we have $f^\beta(\frac{1}{2}) > f^1(\frac{1}{2})$, therefore consumers' locations under f^β are more concentrated around $\frac{1}{2}$ than under f^1; this makes market 2 more competitive than market 1, given the symmetry between the firms in this market. Additionally, we assume that the transportation cost parameter in market 2 is smaller than in market 1, $0 < t_2 \leq t_1 = 1$.

We need to extend the analysis of price competition for the bundle in this asymmetric case. Note that a consumer with location (x_1, x_2) is indifferent between both bundles when

$$\alpha_1 - P_A - t_1 x_1 - t_2 x_2 = -P_B - t_1(1-x_1) - t_2(1-x_2).$$

Let $x = (t_1 x_1 + t_2 x_2)/(t_1 + t_2)$ denote the weighted average location of a consumer. In equilibrium, the weighted average location of the indifferent consumer is implicitly given by

$$x = \frac{1}{2} + \frac{\alpha_1}{2(t_1+t_2)} + \frac{1 - 2\tilde{F}(x)}{\tilde{f}(x)}$$

where $\tilde{F}$ is now the distribution function of the weighted average of x_1 and x_2. In particular, $\tilde{F}$ depends on t_1 and t_2.

Our next proposition describes our numerical results about leverage in this setting.

Proposition 5. *Consider competition between two firms each producing two products. Suppose that s_1 is uniformly distributed over $[0,1]$, while s_2 is distributed according to the β-distribution f^β with $\beta \geq 1$. Let $t_1 = 1 \geq t_2$ and $\alpha_1 > 0 = \alpha_2$. Firm A can profitably leverage its dominance from market 1 by bundling the products when the dominance is strong enough* *and market 2 is sufficiently more competitive. Table 1 reports for different values of β and t_2 cutoff values for α_1 above which bundling is profitable for firm A. Bundling always reduces the profit of firm B.*

$\beta \downarrow \backslash t_2 \rightarrow$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1	2.883	2.833	2.849	2.937	dne	dne	dne	dne	dne	dne
2	2.875	2.810	2.786	2.797	2.843	2.924	dne	dne	dne	dne
3	2.880	2.814	2.780	2.772	2.787	2.824	2.884	2.967	dne	dne
4	2.886	2.821	2.783	2.766	2.768	2.786	2.821	2.872	2.940	dne
5	2.892	2.828	2.788	2.767	2.761	2.769	2.790	2.824	2.872	2.932
6	2.895	2.834	2.794	2.770	2.759	2.761	2.773	2.797	2.831	2.876
7	2.901	2.840	2.799	2.773	2.760	2.757	2.764	2.780	2.806	2.840
8	2.904	2.845	2.805	2.777	2.761	2.755	2.758	2.769	2.788	2.816
9	2.908	2.850	2.809	2.782	2.764	2.755	2.755	2.762	2.777	2.798

Table 1: Asymmetric competition intensity: Cutoff values $\hat{\alpha}_1$ above which bundling is profitable for firm A. The label *dne* indicates that such cutoff does not exist.

Note from Table 1 that whether bundling is profitable for a given level of dominance α_1 is non-monotonic in β and in t_2.

To illustrate the proposition, let us discuss the case of $\beta = 5$ and $t_2 = 1$. Figure 2 shows the density functions of consumers' locations in each market and for the average (bundle). Competition in market 2 is more intense because f^5 is more peaked than the uniform distribution: Without dominance, symmetric equilibrium prices are equal to 1 in market 1 and 0.406 in market 2. Note that the density function corresponding to the bundle is more peaked than the uniform distribution, but not so with respect to f^5. When firm A has dominance level $\alpha_1 = 3$, under independent pricing, $p^*_{A1} = 2$ and $p^*_{B1} = 0$ hold and firm A has 100% market share in market 1. Firm 1 has 50% market share in market 2 as we assume $\alpha_2 = 0$. So firm A produces 75% of all units. Firm A's profit equals 2.203 and firm B's profit equals 0.203.

When firms A and B bundle without changing total prices, A's market share goes up to 94%, indicating that the demand size effect is large. Firm A's profit then also goes up, to about 2.26, because he loses only a small fraction of sales in market 1 (where the profit margin is high), whereas he gains a lot of market share in market 2. In the equilibrium under

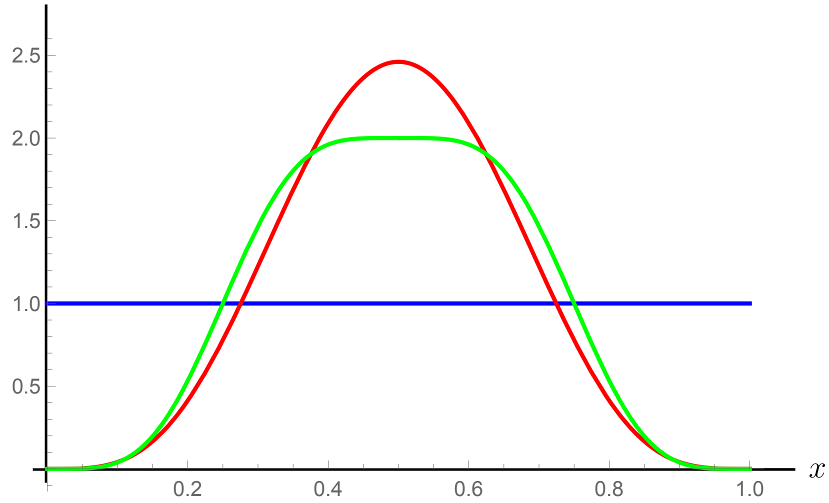

Figure 2: Density functions for market 1 (blue), market 2 (red) and (half) the bundle (green).

bundling, firm B reduces the total price (from 0.406 till 0.325) while firm A increases it (from 2.406 till 2.501), only partially undoing the demand size effect (from 94% till 88.5%).

4.2 Leverage of multiple products

We now assume that A and B compete in $n \geq 3$ different markets, and that A is dominant in k markets, with $1 \leq k \leq n-1$: $\alpha_1 = ... = \alpha_k \equiv \alpha > \alpha_{k+1} = ... = \alpha_n = 0$. We consider symmetric transportation cost $t_1 = ... = t_n = 1$ and uniformly distributed consumer locations in all markets. Then bundling is profitable for firm A when it is dominant enough in sufficiently many markets.

Proposition 6. *Consider competition between two firms each producing $n \geq 3$ products. Suppose that s_j is independently and uniformly distributed over $[0,1]$ for $j = 1, ..., n$. Assume $t_1 = ... = t_n = 1$ and $\alpha_1 = ... = \alpha_k \equiv \alpha \in (0,3]$ and $\alpha_{k+1} = ...\alpha_n = 0$. Then bundling always reduces firm B's profit but in some cases it increases firm A's profit if α is sufficiently close to 3. Table 2 reports for some pairs (n,k) the cutoff value $\hat{\alpha} \in (0,3]$ such that bundling increases the profit of firm A if $\alpha \in (\hat{\alpha}, 3]$.*

$n\downarrow\backslash k\rightarrow$	1	2	3	4	5	6	7	8	9
3	dne	2.496	-	-	-	-	-	-	-
4	dne	dne	2.023	-	-	-	-	-	-
5	dne	dne	2.705	1.794	-	-	-	-	-
6	dne	dne	dne	2.235	1.662	-	-	-	-
7	dne	dne	dne	2.736	1.986	1.573	-	-	-
8	dne	dne	dne	dne	2.337	1.828	1.508	-	-
9	dne	dne	dne	dne	2.723	2.099	1.718	1.458	-
10	dne	dne	dne	dne	dne	2.389	1.937	1.636	1.418

Table 2: Multiple products: Cutoff values $\hat{\alpha}_1$ above which bundling is profitable for firm A when dominant in k out of n markets. The label *dne* indicates that such cutoff does not exist.

5 Competing against specialists

Up to now, we have considered competition between two generalists. In this section, we consider competition between a generalist firm A and specialists B1, ..., Bn where Bj ($j = 1, 2, ..., n$) denotes the firm specialized in product j. For instance, in the case of GE-Honeywell merger, the merged firm would compete against engine specialists and avionics specialists. In the absence of bundling, competition in market j between A and Bj occurs as we have described in Section 3.1.[26]

Under pure bundling of A's products, it is as if the specialists offer a bundle of their products at the price $\sum_{j=1}^{n} p_{Bj}$ and consumers choose between A's bundle and the bundle of the specialists. As each specialist chooses its price non-cooperatively, bundling creates a Cournot complement problem: for any given price of A's bundle, the specialists choose prices too high relative to the price that would be chosen by a generalist firm B, because each specialist does not internalize the negative externality on the demand for the other specialists when it raises its price. This implies that A's profit under bundling is strictly higher when the rival firms are separate than when they are integrated. However, it is not clear whether the rivals' profits under bundling will be lower than when they are integrated.

[26]Denicolo (2000) considers competition between a generalist and two specialists when two firms in each product market are symmetric and characterizes how bundling affects profits when the two products are asymmetrically differentiated.

On the one hand, they set too high prices due to the Cournot complement problem, but on the other hand, this in turn induces A to charge a higher price because prices are strategic complements.

Firm A produces all products and sells them in a bundle at total price np_A. Firms B1,...,Bn produce one good each, sold at prices $p_{B1}, ..., p_{Bn}$. The indifferent consumer has average location y_n where

$$y_n = \frac{1}{2} + \sigma \left(\alpha - p_A + \frac{1}{n} \sum_{j=1}^{n} p_{Bj} \right), \tag{7}$$

with $\sigma = 1/(2t)$. We denote with y_n^{**} the equilibrium average location of the indifferent consumer, and we can argue as in subsection 3.1 to conclude that y_n^{**} is the unique fixed point of the mapping

$$Y_n^\alpha : y \mapsto \frac{1}{2} + \sigma\alpha + \frac{n - (n+1)F_n(y)}{f_n(y)} \tag{8}$$

and y_n^{**} determines the equilibrium prices and profits as described by next proposition.

Proposition 7 (Specialists). *The pure bundling pricing game against specialists has a unique equilibrium, characterized by the unique fixed point $y_n^{**}(\alpha)$ of Y_n^α, and $y_n^{**}(\alpha) \in (\frac{1}{2}, 1)$. The function $y_n^{**}(\alpha)$ is increasing and concave for $\alpha \geq 0$. The equilibrium prices per product are*

$$p_{n,A}^{**}(\alpha) = \frac{F_n(y_n^{**}(\alpha))}{\sigma f_n(y_n^{**}(\alpha))}, \qquad p_{n,Bj}^{**}(\alpha) = \frac{n(1 - F_n(y^{**}(\alpha)))}{\sigma f_n(x^{**}(\alpha))} \quad \textit{for } j = 1, ..., n.$$

The equilibrium profits are

$$\Pi_{n,A}^{**}(\alpha) = \frac{nF_n(y_n^{**}(\alpha))^2}{\sigma f_n(y_n^{**}(\alpha))}, \qquad \Pi_{n,Bj}^{**}(\alpha) = \frac{n(1 - F_n(y_n^{**}(\alpha)))^2}{\sigma f_n(y_n^{**}(\alpha))} \quad \textit{for } j = 1, ..., n.$$

$p_{n,A}^{**}$ and $\Pi_{n,A}^{**}$ are increasing in α while $p_{n,B}^{**}$ and $\Pi_{n,Bi}^{**}$ are decreasing in α.

Note that $Y_n^\alpha(y) - X_n^\alpha(y) = (n-1)(1 - F_n(y))/f_n(y) > 0$ for all $y \in (0, 1)$. Hence, $y_n^{**} > x_n^*$, that is the indifferent consumer is further away from firm A when A competes against specialists than when A faces a generalist opponent. This immediately implies the following results.

Corollary 1 (Cournot complement). *In comparison with the competition in bundles against a generalist, bundling by a generalist who competes against specialists,*

(i) leads to higher prices by both the generalist and the specialists and

(ii) yields the generalist higher sales and profit.

Consider the case where consumers are uniformly distributed. Recall that for $n = 2$, with generalists, bundling is profitable for firm A when $\alpha > 1.415t$ and is profitable for firm B when $\alpha > 2.376t$. In the case of specialists, bundling is profitable for firm A when $\alpha > 0.307t$ and is profitable for each firm Bj when $\alpha > 2.092t$. However, for $n = 3$, with generalists, bundling is profitable for firm A when $\alpha > 1.425t$ and is profitable for firm B when $\alpha > 2.541t$. In the case of specialists, bundling is profitable for firm A when $\alpha > 0.093t$ and is profitable for each firm Bj when $\alpha > 2.595t$. Therefore, when B is separated into specialists, it greatly expands the range of dominance under which bundling is credible and reduces the rivals' profits. Moreover, under bundling, the joint profits of the specialists may be higher than when they are integrated. For instance, when $n = 2$, for $2.092t < \alpha < 2.376t$, the joint profit of the specialists under bundling is higher than their profit under independent pricing, which in turn is higher than the generalist B's profit under bundling.

Gans and King (2006) consider a model with 4 symmetric specialist firms, where a pair of firms can offer a bundled discount. This creates a similar Cournot complement effect, both within the pair offering the discount and the rival pair, increasing the profit of the former and reducing the profit of the latter.

5.1 Leverage when competing against specialists

Suppose now that firm A competes against two specialists as in Section 5. Assume $t = t_1 = t_2 = 1$ and $\alpha_1 > \alpha_2 = 0$. We have:

Proposition 8. *Consider competition between a generalist firm A and two specialists B1 and B2. Suppose that* (s_1, s_2) *is uniformly distributed over* $[0, 1]^2$ *and assume* $t_1 = t_2 = 1$ *and* $\alpha_1 \in (0, 3)$ *and* $\alpha_2 = 0$*. Then, pure bundling always decreases B2's profit and the joint profit of B1 and B2. In addition, for* $\alpha_1 > 0.701$*, pure bundling increases A's profit; for* $\alpha_1 > 1.159$*, pure bundling increases B1's profit.*

The proposition should be contrasted with the fact that if B1 and B2 are integrated, tying is never profitable for A for $\alpha_1 \in (0, 3)$ and $\alpha_2 = 0$. Therefore, the proposition essentially captures the Cournot complement effect which arises when the competing firms are separated. Then, for $\alpha_1 > 0.701$, tying is profitable and reduces both B2's profit and the joint profit of B1 and B2.

6 Conclusion

We contribute to the leverage theory of tying by studying the leverage of a dominant firm instead of a pure monopolist in the tying product market. We find that the dominant firm benefits from a positive demand size effect of bundling, which makes its bundling credible as long as the demand elasticity effect is not too negative. By contrast, in the case of a pure monopolist, we find that both the demand size effect and the demand elasticity effect are negative, which makes its bundling not credible.[27] We identify four different factors that allow a firm to profitably leverage its dominance in tying product markets to tied product markets where it is not dominant: asymmetric distribution of consumers, asymmetric transportation costs, multiple dominant products, competition against specialists. In reality, some of these forces can coexist. For instance, when a channel conglomerate bundles several strong channels with some weak ones against multiple small rivals, the last two forces are combined to make bundling credible and powerful. Our findings provide a justification for the use of contractual bundling for foreclosure purposes.

A Appendix

A.1 Proof of Proposition 1

(i) The condition of the proposition implies that $\lim_{x\to 1} X^{\alpha}(x) < 1$, so that the fixed point $x^*(\alpha)$ is smaller than 1.

Now we show that $x^*(\alpha)$ is increasing and concave for $\alpha \geq 0$. By taking the derivative w.r.t. α on both sides of the equation $X^{\alpha}(x^*(\alpha)) = x^*(\alpha)$, one obtains immediately

$$\frac{dx^*(\alpha)}{d\alpha} = \frac{\sigma}{3 + \frac{1-2F(x^*(\alpha))}{f(x^*(\alpha))}\frac{f'(x^*(\alpha))}{f(x^*(\alpha))}}.$$

Moreover, it follows that $\frac{dx^*(\alpha)}{d\alpha}$ is a strictly positive and weakly decreasing function of α: First note that both $(1-2F(x))/f(x)$ and $f'(x)/f(x)$ are non-positive for $x \geq 1/2$. Next observe that both functions are decreasing because of log-concavity of f. The product of these two functions is thus positive and increasing. Since $x^*(\alpha)$ is increasing in α, it follows that $\frac{dx^*(\alpha)}{d\alpha}$ is decreasing.

[27]See our working paper Hurkens, Jeon and Menicucci (2016) for the analysis.

Next, the equilibrium price of firm A is increasing in α because (i) $x^*(\alpha)$ is increasing in α and (ii) $F(x)/f(x)$ is increasing in x by log-concavity. The equilibrium profit of firm A is then also increasing because both equilibrium price (as seen above) and market share ($F(x^*(\alpha))$) are increasing. Similarly, the equilibrium price and profit for firm B are decreasing.

(ii) In this case, no interior equilibrium exists. Necessarily, $p_B^* = 0$ and firm A corners the market. The highest price to corner the market, given $p_B^* = 0$, is $p_A^* = \alpha - 1/(2\sigma)$. Clearly firm A has no incentive to set a lower price (as demand cannot be increased). Firm A has also no incentive to increase its price because the marginal profit, evaluated at p_A^*, equals

$$\frac{d\pi_A}{dp_A} = F(1) - \sigma p_A^* f(1) \leq 0,$$

where the inequality follows directly from $(\sigma\alpha - 1/2)f(1) \geq 1$. By virtue of the remark in footnote 13, it follows that $\frac{d\pi_A}{dp_A} < 0$ for any $p_A > p_A^*$. ■

A.2 Proof of Proposition 2

The proof is similar to that of Proposition 1 and therefore omitted. ■

A.3 Proof of Lemma 1

Proof of (i) We first show the result for $n = 2$. Note that

$$f_2(1/2) = 2\int_0^1 f(s)^2 ds = 4\int_0^{1/2} f(s)^2 ds > 4\int_{1/4}^{1/2} f(s)^2 ds. \tag{9}$$

Next, observe that for a log-concave function f we have $\log[f(a)f(b)] = \log[f(a)] + \log[f(b)] \leq 2\log[f(\frac{a+b}{2})] = \log[f(\frac{a+b}{2})^2]$ for any a and b in $(0,1)$, hence

$$f(a)f(b) \leq f(\frac{a+b}{2})^2. \tag{10}$$

In particular, taking $b = 1/2$ and $a = 2s - 1/2$ in (10) yields $f(s)^2 \geq f(2s - 1/2)f(1/2)$ for $s > 1/4$. And thus

$$\int_{1/4}^{1/2} f(s)^2 ds \geq f(1/2)\int_{1/4}^{1/2} f(2s - 1/2) ds = f(1/2)\int_0^{1/2} \frac{1}{2} f(y) dy = f(1/2)/4.$$

Combining this with (9) we obtain $f_2(1/2) > f(1/2)$.

In order to prove the result for $n > 2$, recall that F_j is more peaked than F_{j-1} for each $j \geq 2$, hence $F_j(x) > F_{j-1}(x)$ for each $x \in (\frac{1}{2}, 1)$. Since $F_j(1/2) = F_{j-1}(1/2)$, it follows that $f_j(\frac{1}{2}) \geq f_{j-1}(\frac{1}{2})$. Hence, we conclude that $f_j(1/2) \geq f_2(1/2) > f(1/2)$ for each $j \geq 2$.

Proof of (ii) We denote the i-th derivative of a function g by $g^{(i)}$, such that $g^{(0)} = g$. For $j \geq 2$, let $k_j \geq 1$ be such that $f_j^{(i)}(1) = 0$ for $i = 0, 1, \ldots, k_j - 1$ and $f_j^{(k_j)}(1) \neq 0$. Regarding $j = 1$, we define k_1 as k_j above if $f(1) = 0$; we define $k_1 = 0$ if $f(1) > 0$. We will prove that $k_{j+1} = k_j + k_1 + 1$ for all $j \geq 1$. By L'Hôpital's rule this implies $\lim_{x\uparrow 1} f_{j+1}(x)/f_j(x) = 0$ for each $j \geq 1$, hence $\lim_{x\uparrow 1} \frac{f_n(x)}{f(x)} = 0$.

We have that

$$f_{j+1}(x) = \int_{(j+1)x-j}^{1} \frac{j+1}{j} f(s) f_j\left(\frac{(j+1)x - s}{j}\right) ds$$

hence

$$f_{j+1}^{(i)}(x) = \int_{(j+1)x-j}^{1} \left(\frac{j+1}{j}\right)^{i+1} f(s) f_j^{(i)}\left(\frac{(j+1)x - s}{j}\right) ds$$

and $f_{j+1}^{(i)}(1) = 0$ for $i = 1, ..., k_j$. However, for $m \geq 1$ we find

$$\begin{aligned} f_{j+1}^{(k_j+m)}(x) \quad = \quad & -\sum_{h=1}^{m} (j+1)^{m-h+1} \left(\frac{j+1}{j}\right)^{k_j+h} f^{(m-h)}((j+1)x - j) f_j^{(k_j+h-1)}(1) + \\ & \int_{(j+1)x-j}^{1} \left(\frac{j+1}{j}\right)^{k_j+m+1} f(s) f_j^{(k_j+m)}\left(\frac{(j+1)x - s}{j}\right) ds \end{aligned}$$

Hence,

$$f_{j+1}^{(k_j+m)}(1) = -\sum_{h=1}^{m} (j+1)^{m-h+1} \left(\frac{j+1}{j}\right)^{k_j+h} f^{(m-h)}(1) f_j^{(k_j+h-1)}(1)$$

and $f_{j+1}^{(k_j+m)}(1) = 0$ if $m \leq k_1$, but $f_{j+1}^{(k_j+k_1+1)}(1) = -(j+1)^{k_1+1} \left(\frac{j+1}{j}\right)^{k_j+1} f^{(k_1)}(1) f_j^{(k_j)}(1) \neq 0$. Therefore $k_{j+1} = k_j + k_1 + 1$. ■

A.4 Proof of Lemma 2

We start by defining several dominance level sets.

Definition 1. (i) $\mathcal{A}_{MS}^{+} = \{\alpha \geq 0 \ : \ F_n(x_n^*(\alpha)) \geq F(x^*(\alpha))\}$

(ii) $\mathcal{A}_{\pi A}^{+} = \{\alpha \geq 0 \ : \ \Pi_{n,A}^*(\alpha) \geq n\pi_A^*(\alpha)\}$

(iii) $\mathcal{A}_{\pi B}^{+} = \{\alpha \geq 0 \ : \ \Pi_{n,B}^*(\alpha) \geq n\pi_B^*(\alpha)\}$

(iv) $\mathcal{A}^+_{DENS} = \{\alpha \geq 0 : \; f_n(x^*_n(\alpha)) \geq f(x^*(\alpha))\}$

(v) $\mathcal{A}^+_{PA} = \{\alpha \geq 0 : \; P^*_{n,A}(\alpha) \geq np^*_A(\alpha)\}$

(vi) $\mathcal{A}^+_{PB} = \{\alpha \geq 0 : \; P^*_{n,B}(\alpha) \geq np^*_B(\alpha)\}$
Let furthermore $\mathcal{A}^-_K = [0,\infty) \setminus \mathcal{A}^+_K$ *for* $K \in \{MS, \pi A, \pi B, DENS\}$

Lemma 2 is equivalent to the following strict superset relations between various dominance level sets:

$$\mathcal{A}^+_{\pi A} \supset \mathcal{A}^+_{\pi B} \supset \mathcal{A}^-_{MS}. \tag{11}$$

We below prove (11). The proof consists of several steps. In step 1 we prove that both firms gain from bundling for dominance levels for which firm A obtains lower market share under bundling. In step 2 we prove the weak set relations of (11) for dominance levels for which firm A obtains higher market share under bundling. In step 3 we prove the strictness of the set relations. This is easy in case $f(1) > 0$, but requires additional steps 3.1 and 3.2 in case $f(1) = 0$.

Step 1. We first establish that if the dominance level belongs to $\mathcal{A}^-_{MS}$, then both firms will set higher total prices and obtain higher profits under bundling. Let $\bar{\alpha} \in \mathcal{A}^-_{MS}$ be such a dominance level, that is $F_n(x^*_n(\bar{\alpha})) < F(x^*(\bar{\alpha}))$. As the distribution of the average location is more peaked, this implies that $x^*(\bar{\alpha}) > x^*_n(\bar{\alpha})$. From (2) and (5) we know that for any α such that $(\sigma\alpha - 1/2)f(1) < 1$ we have[28]

$$x^*_n(\alpha) - x^*(\alpha) = \frac{1 - 2F_n(x^*_n(\alpha))}{f_n(x^*_n(\alpha))} - \frac{1 - 2F(x^*(\alpha))}{f(x^*(\alpha))}. \tag{12}$$

In particular, for $\bar{\alpha}$ the left-hand side of (12) is negative. Eq. (12) can then only hold if $f_n(x^*_n(\bar{\alpha})) < f(x^*(\bar{\alpha}))$. That is, $\bar{\alpha} \in \mathcal{A}^-_{DENS}$. Using the expressions for equilibrium prices of firm B from Propositions 1 and 2 we conclude that $P^*_{n,B}(\bar{\alpha}) > np^*_B(\bar{\alpha})$. As firm B also obtains higher market share under bundling, firm B's profit is higher under bundling as well. Now firm A could set bundle price $P_{n,A} = np^*_A$ and obtain higher market share, and thus higher profits than what he obtains in the independent pricing equilibrium. The optimal bundle price for firm A yields at least as much profit. As we know that in equilibrium firm A obtains less market share than under independent pricing, the optimal bundle price must be such that $P^*_{n,A}(\bar{\alpha}) > np^*_A(\bar{\alpha})$.

[28] If $(\sigma\bar{\alpha} - 1/2)f(1) \geq 1$, then Proposition 1(ii) applies and thus $\Pi^*_{n,B}(\bar{\alpha}) > n\pi^*_B(\bar{\alpha}) = 0$, $P^*_{n,B}(\bar{\alpha}) > np^*_B(\bar{\alpha}) = 0$. Minor changes to the arguments below establish that $\Pi^*_{n,A}(\bar{\alpha}) > n\pi^*_A(\bar{\alpha})$, $P^*_{n,A}(\bar{\alpha}) > np^*_A(\bar{\alpha})$.

Step 2. Next we focus on dominance levels for which firm A obtains higher market share under bundling, that is $\alpha \in \mathcal{A}^+_{MS}$. We will show that

$$(\mathcal{A}^+_{\pi A} \cap \mathcal{A}^+_{MS}) \supseteq (\mathcal{A}^+_{PA} \cap \mathcal{A}^+_{MS}) \supseteq (\mathcal{A}^-_{DENS} \cap \mathcal{A}^+_{MS}) \supseteq (\mathcal{A}^+_{PB} \cap \mathcal{A}^+_{MS}) \supseteq (\mathcal{A}^+_{\pi B} \cap \mathcal{A}^+_{MS}).$$

It is straightforward that $\mathcal{A}^+_{\pi A} \cap \mathcal{A}^+_{MS} \supseteq \mathcal{A}^+_{PA} \cap \mathcal{A}^+_{MS}$. Namely, for dominance levels for which firm A sets higher total price and obtains higher market share under bundling, profits are automatically higher under bundling.

Note that for $\alpha \in \mathcal{A}^-_{DENS} \cap \mathcal{A}^+_{MS}$, $P^*_{n,A} = nF_n(x^*_n(\alpha))/(\sigma f_n(x^*_n(\alpha))) > nF(x^*(\alpha))/(\sigma f(x^*(\alpha))) = np^*_A$, because the numerator is larger (and positive) and the denominator is strictly smaller (but positive) on the left-hand side. This shows that $\mathcal{A}^+_{PA} \cap \mathcal{A}^+_{MS} \supseteq \mathcal{A}^-_{DENS} \cap \mathcal{A}^+_{MS}$.

Note that for $\alpha \in \mathcal{A}^+_{DENS} \cap \mathcal{A}^+_{MS}$, $P^*_{n,B} = n(1 - F_n(x^*_n(\alpha)))/(\sigma f_n(x^*_n(\alpha))) \leq n(1 - F(x^*(\alpha)))/(\sigma f(x^*(\alpha))) = np^*_B$, because the numerator is smaller (and positive) and the denominator is larger (and positive) on the left-hand side. Moreover, the inequality must be strict. Namely, the inequality could only be binding when both $f(x^*(\alpha)) = f_n(x^*_n(\alpha))$ and $F(x^*(\alpha)) = F(x^*_n(\alpha))$. But this would imply that $x^*(\alpha) = x^*_n(\alpha)$ because of (12). However, this is incompatible with $F(x^*(\alpha)) = F_n(x^*_n(\alpha))$ because $F_n(x) > F(x)$ for any $x \in (1/2, 1)$. This proves that $\mathcal{A}^-_{DENS} \cap \mathcal{A}^+_{MS} \supseteq \mathcal{A}^+_{PB} \cap \mathcal{A}^+_{MS}$.

It is straightforward that $\mathcal{A}^+_{PB} \cap \mathcal{A}^+_{MS} \supseteq \mathcal{A}^+_{\pi B} \cap \mathcal{A}^+_{MS}$. Namely, for dominance levels for which firm B obtains higher total profit under bundling, despite having smaller market share under bundling, it must be that the total price is higher under bundling.

Step 3. We show that the set relations are strict. We know that $f_n(x^*_n(0)) > f(x^*(0))$, but if $f(1) > 0$ and $\alpha \geq 1/(\sigma f(1)) + 1/(2\sigma)$ then necessarily $f_n(x^*_n(\alpha)) < f(1) = f(x^*(\alpha))$.[29] There must then exist a level $\alpha_{DENS} > 0$ for which $f_n(x^*_n(\alpha_{DENS})) = f(x^*(\alpha_{DENS}))$. In the hypothetical case that there exist multiple such levels, we choose the maximal one. We claim that $\alpha_{DENS} \in \mathcal{A}^+_{MS} \cap \mathcal{A}^+_{\pi A} \cap \mathcal{A}^-_{\pi B}$.

It is clear that $\alpha_{DENS} \in \mathcal{A}^+_{MS}$. Namely, suppose it is not true. Then firm B has strictly higher market share under bundling, and thus both firms would obtain higher profits under bundling (from Step 1). However, $f(x^*(\alpha_{DENS})) = f_n(x^*_n(\alpha_{DENS}))$ and $F(x^*(\alpha_{DENS})) > F_n(x^*_n(\alpha_{DENS}))$ contradict $\Pi^*_{n,A}(\alpha_{DENS}) > n\pi^*_A(\alpha_{DENS})$ (from Propositions 1 and 2.).

Using again Propositions 1 and 2, it easily follows that $\alpha_{DENS} \in \mathcal{A}^+_{\pi A}$. It must also be true that firm B has strictly lower profits under bundling. Namely, profits for firm B could

[29]We show below in Step 3.2 that if $f(1) = 0$, then $f_n(x^*_n(\alpha)) < f(x^*(\alpha))$ still holds for a large α.

at best be equal under bundling, but this would require that firm B's market share is exactly the same under both pricing regimes. We have already seen before that this is impossible as it would imply that $x_n^*(\alpha_{DENS}) = x^*(\alpha_{DENS})$ by Eq. (12), and thus $F_n(x) = F(x)$ for $x = x_n^*(\alpha_{DENS})$. This thus proves the strictness of the first superset relation.

In order to prove the second, let $\alpha_{MS} > 0$ be such that market shares of the two firms are equal under both regimes, that is, $F(x^*(\alpha_{MS})) = F_n(x^*(\alpha_{MS}))$. Such a level exists if $f(1) > 0$ as firm B has a strictly lower market share under bundling for small positive dominance levels, while for $\alpha \geq 1/(\sigma f(1)) + 1/(2\sigma)$ his market share is zero under independent pricing (Prop. 1) but positive under bundling (Prop. 2).[30] In the hypothetical case that there exist multiple levels of dominance with this property, we choose the maximal one. We claim that $\alpha_{MS} > \alpha_{DENS}$, and therefore $f_n(x_n^*(\alpha_{MS})) < f(x^*(\alpha_{MS}))$, which implies $\alpha_{MS} \in \mathcal{A}_{\pi B}^+$ even though $\alpha \notin \mathcal{A}_{MS}^-$

This follows easily from, on the one hand, observing that $\alpha_{MS} = \alpha_{DENS}$ leads to a contradiction, as again by Eq. (12) we would deduce that $x_n^*(\alpha_{MS}) = x^*(\alpha_{MS})$, which is impossible. On the other hand, $\alpha_{MS} < \alpha_{DENS}$ is impossible because of the assumption that α_{MS} has been chosen as the maximal level of dominance for which market shares are equal under the two regimes. It implies that for higher levels, in particular for α_{DENS}, market share is strictly lower for firm A under bundling. But we have already established before that $\alpha_{DENS} \in \mathcal{A}_{MS}^+$. ■

Step 3.1 Suppose that $f(1) = 0$. We prove that for $j = 1, ..., n-1$, we have $F_j(x_j^*(\alpha)) > F_{j+1}(x_{j+1}^*(\alpha))$ for large α. It then follows that for large α, $F(x^*(\alpha)) > F_n(x_n^*(\alpha))$.
Define the strictly increasing function $W_j(x) = x + \frac{2F_j(x)-1}{f_j(x)}$. Given $\alpha > 0$, we know that $x_j^*(\alpha)$ is such that $W_j(x_j^*(\alpha)) = \frac{1}{2} + \sigma\alpha$. For a large α, both $x_j^*(\alpha)$ and $x_{j+1}^*(\alpha)$ are close to 1. Thus, given x close to 1, we select $y(x)$ as the unique y such that $F_{j+1}(y) = F_j(x)$. We prove that $W_{j+1}(y(x)) > W_j(x)$ for x close to 1, hence $W_{j+1}(y(x_j^*(\alpha))) > W_j(x_j^*(\alpha)) = \frac{1}{2} + \sigma\alpha$ for a large α, which implies $x_{j+1}^*(\alpha) < y(x_j^*(\alpha))$ and thus $F_{j+1}(x_{j+1}^*(\alpha)) < F_{j+1}(y(x_j^*(\alpha))) = F_j(x_j^*(\alpha))$ for large α.

In order to prove $W_{j+1}(y(x)) > W_j(x)$, we notice that $F_{j+1}(y(x)) = F_j(x)$ makes the inequality equivalent to $(y(x)-x)f_{j+1}(y(x)) + (2F_j(x)-1)[1 - \frac{f_{j+1}(y(x))}{f_j(x)}] > 0$. We prove that $\lim_{x\uparrow 1} \frac{f_{j+1}(y(x))}{f_j(x)} = 0$, hence $\lim_{x\uparrow 1} \left((y(x)-x)f_{j+1}(y(x)) + (2F_j(x)-1)[1 - \frac{f_{j+1}(y(x))}{f_j(x)}]\right) = 1$.

As in the proof of Lemma 1, for $\ell = 1, ..., n$ we let $k_\ell \geq 1$ be such that $f_\ell^{(i)}(1) = 0$ for

[30] We show below in Step 3.1 that if $f(1) = 0$, then $F(x^*(\alpha)) > F_n(x_n^*(\alpha))$ still holds for a large α.

$i = 0, 1, \ldots, k_\ell - 1$ and $f_\ell^{(k_\ell)}(1) \neq 0$. Furthermore, we set $a_\ell = (-1)^{k_\ell} f_\ell^{(k_\ell)}(1)/(k_\ell!) > 0$ and $b_\ell = a_\ell/(k_\ell + 1) > 0$, such that $b_\ell < a_\ell$. Then use Taylor's formula to obtain

$$f_\ell(x) = a_\ell(1-x)^{k_\ell} + \eta_{f_\ell}(x) \tag{13}$$

$$1 - F_\ell(x) = b_\ell(1-x)^{k_\ell+1} + \eta_{F_\ell}(x) \tag{14}$$

with

$$\lim_{x\uparrow 1} \frac{\eta_{f_\ell}(x)}{(1-x)^{k_\ell}} = \lim_{x\uparrow 1} \frac{\eta_{F_\ell}(x)}{(1-x)^{k_\ell+1}} = 0 \tag{15}$$

Let $\varepsilon > 0$ be close enough to zero to satisfy $\varepsilon < b_\ell < a_\ell$ for $\ell = 1, ..., n$, and let $\delta > 0$ be such that for all $x \in (1-\delta, 1)$ we have

$$|\eta_{f_\ell}(x)| < \varepsilon(1-x)^{k_\ell} \tag{16}$$

$$|\eta_{F_\ell}(x)| < \varepsilon(1-x)^{k_\ell+1} \tag{17}$$

Of course, if x is close to 1 then both x and $y(x)$ belong to $(1-\delta, 1)$. From (14) and (17) with $\ell = j+1$ we obtain

$$(b_{j+1} - \varepsilon)(1-y)^{k_{j+1}+1} < 1 - F_{j+1}(y) < (b_{j+1} + \varepsilon)(1-y)^{k_{j+1}+1}$$

Therefore $y(x)$, the solution to $F_{j+1}(y) = F_j(x)$, satisfies

$$\frac{1 - F_j(x)}{b_{j+1} + \varepsilon} < (1 - y(x))^{k_{j+1}+1} < \frac{1 - F_j(x)}{b_{j+1} - \varepsilon} \tag{18}$$

Similarly, from (14) and (17) (with $\ell = j$) we obtain

$$(b_j - \varepsilon)(1-x)^{k_j+1} < 1 - F_j(x) < (b_j + \varepsilon)(1-x)^{k_j+1} \tag{19}$$

Combining (18) and (19) we thus conclude that

$$\frac{b_j - \varepsilon}{b_{j+1} + \varepsilon}(1-x)^{k_j+1} < (1 - y(x))^{k_{j+1}+1} < \frac{b_j + \varepsilon}{b_{j+1} - \varepsilon}(1-x)^{k_j+1} \tag{20}$$

Similarly, using (13) and (16) we obtain

$$\frac{f_{j+1}(y(x))}{f_j(x)} < \frac{(a_{j+1} + \varepsilon)(1 - y(x))^{k_{j+1}}}{(a_j - \varepsilon)(1-x)^{k_j}} \tag{21}$$

We conclude that

$$\frac{f_{j+1}(y(x))}{f_j(x)} < \left(\frac{a_{j+1} + \varepsilon}{a_j - \varepsilon}\right) \left(\frac{b_j + \varepsilon}{b_{j+1} - \varepsilon}\right)^{\frac{k_{j+1}}{k_{j+1}+1}} (1-x)^{\frac{k_{j+1} - k_j}{k_{j+1}+1}}$$

which proves that

$$\lim_{x\uparrow 1}\frac{f_{j+1}(y(x))}{f_j(x)}=0$$

because $k_{j+1}>k_j$. ■

Step 3.2 Suppose that $f(1)=0$. We prove that for $j=1,...,n-1$, we have $f_{j+1}(x_{j+1}^*(\alpha))<f_j(x_j^*(\alpha))$ if $f(1)=0$ and α is large. It then follows that for large α, $f_n(x_n^*(\alpha))<f(x^*(\alpha))$.

Given x close to 1, we select $z(x)$ as the unique $z\in(\frac{1}{2},x)$ such that $f_{j+1}(z)=f_j(x)$. We prove that $W_{j+1}(z(x))<W_j(x)$ for x close to 1, hence $W_{j+1}(z(x_j^*(\alpha)))<W_j(x_j^*(\alpha))=\frac{1}{2}+\sigma\alpha$ for a large α, which implies $x_{j+1}^*(\alpha)>z(x_j^*(\alpha))$ and thus $f_{j+1}(x_{j+1}^*(\alpha))<f_{j+1}(z(x_j^*(\alpha)))=f_j(x_j^*(\alpha))$. The inequality $W_{j+1}(z(x))<W_j(x)$ reduces to $f_j(x)(z(x)-x)<2[F_j(x)-F_{j+1}(z(x))]$, and since $z(x)<x$ it suffices to prove that $F_j(x)>F_{j+1}(z(x))$. We know from the proof of Step 3.1 that if the equality $F_{j+1}(z(x))=F_j(x)$ holds for x close to 1, then $f_{j+1}(z(x))<f_j(x)$. In order to obtain $f_{j+1}(z(x))=f_j(x)$ it is necessary to decrease $z(x)$, which implies $F_{j+1}(z(x))<F_j(x)$. ■

A.5 Proof of Proposition 3

(i) Define $\underline{\alpha}=\min\mathcal{A}_{\pi A}^+$, $\alpha_{\pi A}=\sup\mathcal{A}_{\pi A}^-$, $\alpha_{\pi B}=\min\mathcal{A}_{\pi B}^+$ and $\bar{\alpha}=\sup\mathcal{A}_{\pi B}^-$.

(ii) This follows straightforwardly from (i). ■

A.6 Proof of Proposition 4

(i) In the market for a single product j, for a given location x of the indifferent consumer, social welfare under IP is given by $W(x)=\alpha F(x)-T(x)$ (omitting v_{Bj}), where $T(x)=t\int_0^x zf(z)dz+t\int_x^1(1-z)f(z)dz$ is the total transportation cost incurred by all consumers in that market, and is increasing for $x>\frac{1}{2}$. Likewise, under PB, given the location x of the indifferent consumer, social welfare per product is given by $W_n(x)=\alpha F_n(x)-T_n(x)$, where $T_n(x)=t\int_0^x zf_n(z)dz+t\int_x^1(1-z)f_n(z)dz$. Both W and W_n are maximized by the same location $x_w=\min\{\frac{1}{2}+\sigma\alpha,1\}$, but the location of the equilibrium indifferent consumer is smaller than x_w. Precisely, the functions X^α and X_n^α introduced in (2) and in (5), respectively, are such that $X^\alpha(\frac{1}{2}+\sigma\alpha)<\frac{1}{2}+\sigma\alpha$ and $X_n^\alpha(\frac{1}{2}+\sigma\alpha)<\frac{1}{2}+\sigma\alpha$. This implies that $x^*(\alpha)<x_w$ and $x_n^*(\alpha)<x_w$ for each $\alpha>0$, except for the case of competition under IP and $(\sigma\alpha-\frac{1}{2})f(1)\geq 1$, because then $x_w=1$ and $x^*(\alpha)=1$.

(ii) Here we prove that $W(x^*(\alpha)) > W_n(x_n^*(\alpha))$ when $\alpha \geq 0$ is close to zero. First notice that $x^*(\alpha)$ and $x_n^*(\alpha)$ are both close to $\frac{1}{2}$,[31] and it is simple to see that $T_n(\frac{1}{2}) > T(\frac{1}{2})$ since consumers cannot mix and match; hence $T_n(x_n^*(\alpha)) > T(x^*(\alpha))$. Combining this with the fact that $\alpha F_n(x_n^*(\alpha))$ and $\alpha F(x^*(\alpha))$ are both about 0 reveals that $W(x^*(\alpha)) > W_n(x_n^*(\alpha))$ when $\alpha \geq 0$ is close to zero. When α is large, we have noticed in the proof to part (i) that $x^*(\alpha) = x_w = 1$, hence $W(x^*(\alpha)) > W_n(x_n^*(\alpha))$ holds again.

For the case of the uniform distribution with $n = 2$ (with $t = 1$), we know from Subsection 3.3 that $x^*(\alpha) = \frac{3+\alpha}{6}$ and $x_2^*(\alpha) = \left(7 + \alpha - \sqrt{9 - 2\alpha + \alpha^2}\right)/8$, hence

$$W(x^*(\alpha)) = \alpha \cdot x^*(\alpha) - \int_0^{x^*(\alpha)} zdz - \int_{x^*(\alpha)}^1 (1-z)dz = -\frac{1}{4} + \frac{1}{2}\alpha + \frac{5}{36}\alpha^2$$

$$\begin{aligned} W_2(x_2^*(\alpha)) &= \alpha\left(1 - 2(1 - x_2^*(\alpha))^2\right) - \int_0^{1/2} z \cdot 4zdz - \int_{1/2}^{x_2^*(\alpha)} z \cdot 4(1-z)dz \\ &\quad - \int_{x_2^*(\alpha)}^1 (1-z)\cdot 4(1-z)dz \\ &= -\frac{1}{3} + \frac{3}{4}\alpha + \frac{1}{8}\alpha^2 - \frac{1}{24}\alpha^3 - \frac{1}{24}\alpha\left(2-\alpha\right)\sqrt{\alpha^2 - 2\alpha + 9} \end{aligned}$$

Numerical analysis reveals that $W_2(x_2^*(\alpha)) > W(x^*(\alpha))$ if and only $\alpha \in (1.071, 2.306)$.

A.7 Proof of Proposition 7

Given y_n defined in (7), the profit functions are

$$\pi_A = p_A n F_n(y_n), \qquad \pi_{Bj} = p_{Bj}(1 - F_n(y_n)) \quad \text{for } j = 1, ..., n$$

and the first-order conditions are

$$0 = F_n(y_n) - \sigma p_A f_n(y_n), \qquad 0 = 1 - F_n(y_n) - \frac{\sigma}{n} p_{Bj} f_n(y_n) \quad \text{for } j = 1, ..., n$$

Given the equilibrium average location y_n^{**}, if $p_{n,A}^{**}, p_{n,B}^{**}$ are the (symmetric) equilibrium prices then $p_{n,A}^{**} = F_n(y_n^{**})/(\sigma f_n(y_n^{**}))$ and $p_{n,B}^{**} = n(1 - F_n(y_n^{**}))/(\sigma f_n(y_n^{**}))$. Hence

$$y_n^{**} = \frac{1}{2} + \sigma\alpha - \sigma(p_{n,A}^{**} - p_{n,B}^{**}) = \frac{1}{2} + \sigma\alpha + \frac{n - (n+1)F_n(y_n^{**})}{f_n(y_n^{**})}.$$

that is y_n^{**} is the fixed point of (8).Notice that $\frac{n-(n+1)F_n(y)}{f_n(y)} = -\frac{F_n(y)}{f_n(y)} + n\frac{1-F_n(y)}{f_n(y)}$, hence Y_n^α is weakly decreasing. Moreover, $Y_n^\alpha(1/2) > 1/2$, $\lim_{y\uparrow 1} Y_n^\alpha(y) = -\infty$, therefore a unique fixed point exists for Y_n^α in the interval $(\frac{1}{2}, 1)$.

[31] From Subsection 3.3 we also know that $\frac{dx^*}{d\alpha}$ and $\frac{dx_n^*}{d\alpha}$ are both equal to $\frac{\sigma}{3}$ when $\alpha = 0$.

B On-line appendix [NOT FOR PUBLICATION]

B.1 Proof of Proposition 8

From the example immediately after Proposition 3, we see that under independent pricing the profit is $\pi^*_{A1}(\alpha_1) + \pi^*_{A2}(0) = 2(\frac{3+\alpha_1}{6})^2 + \frac{1}{2}$ for firm A, and $\pi^*_{B1}(\alpha_1) = 2(\frac{3-\alpha_1}{6})^2$ for firm $B1$, $\pi^*_{B2}(0) = \frac{1}{2}$ for firm $B2$.

In order to find the profit under bundling, we can rely on Proposition 7 and find that $\Pi^{**}_{2,A}(\alpha_1/2) = \frac{4F_2(y_2^{**}(\alpha_1/2))^2}{f_2(y_2^{**}(\alpha_1/2))}$, $\Pi^{**}_{2,B1}(\alpha_1/2) = \Pi^{**}_{2,B2}(\alpha_1/2) = \frac{4(1-F_2(y_2^{**}(\alpha_1/2)))^2}{f_2(y_2^{**}(\alpha_1/2))}$, and $y_2^{**}(\alpha_1/2)$ is the fixed point of the function $Y_2^{\alpha_1/2}(y) = \frac{1}{2} + \frac{1}{2}\frac{\alpha_1}{2} + \frac{2-3\left(1-2(1-y)^2\right)}{4(1-y)}$ in the interval $(\frac{1}{2}, 1)$, that is $y_2^{**}(\alpha_1/2) = \frac{1}{20}\alpha_1 + \frac{9}{10} - \frac{1}{20}\sqrt{\alpha_1^2 - 4\alpha_1 + 44}$. Numerical computations show the result. ■

B.2 Mixed Bundling in the baseline model

Here, we consider the baseline model with $n = 2$ and study the case in which each firm is allowed to practice mixed bundling. This means that firm i $(= A, B)$ chooses a price P_i for the bundle of its own products and a price $p_i = p_{ij}$ for each single product $j = 1, 2$. Thus each consumer buys the bundle of a firm i and pays P_i, or buys one object from each firm and pays $p_A + p_B$. The main result is that when α is sufficiently large, we find the same equilibrium outcome described by Proposition 2 under pure bundling because for firm A a pure bundling strategy is superior to any alternative strategy when it has a large advantage over firm B. Moreover, we show that the same result holds when A competes with specialists B1 and B2.

Without loss of generality, we assume that $P_i \leq 2p_i$ holds for $i = A, B$ and that each consumer willing to buy both products of i buys i's bundle. As a consequence, each consumer chooses one alternative among AA, AB, BA, BB, where for instance AB means buying products $A1$ and $B2$. In order to describe the preferred alternative of each type of consumer, we introduce

$$s' \equiv \frac{1}{2} + \frac{\alpha + P_B - p_A - p_B}{2t} \text{ and } s'' \equiv \frac{1}{2} + \frac{\alpha + p_A + p_B - P_A}{2t}$$

where $s' \leq s''$ holds from $P_A \leq 2p_A$ and $P_B \leq 2p_B$.[32]

We find:

[32]Precisely, s' is such that a consumer located at $(s_1, s_2) = (s', 1)$ (at $(s_1, s_2) = (1, s')$) is indifferent between the alternatives BB and AB (between the alternatives BB and BA). Likewise, s'' is such that a

- Type (s_1, s_2) buys AA if and only if $s_1 \le s''$, $s_2 \le s''$, $s_1 + s_2 \le s' + s''$.
- Type (s_1, s_2) buys AB if and only if $s_1 \le s'$, $s_2 > s''$.
- Type (s_1, s_2) buys BA if and only if $s_1 > s''$, $s_2 \le s'$.
- Type (s_1, s_2) buys BB if and only if $s_1 > s'$, $s_2 > s'$, $s_1 + s_2 > s' + s''$.

Let $S_{ii'}$ and $\mu_{ii'}$ denote, respectively, the set of types who choose ii' and the measure of $S_{ii'}$ for $ii' = AA$, AB, BA, BB. Note that $\mu_{AB} = \mu_{BA}$, and moreover $\mu_{AB} > 0$ if $0 < s'$ and $s'' < 1$;[33] $\mu_{AB} = 0$ (as in Section 3.2) if $s' \le 0$ and/or $s'' \ge 1$.[34] In either case, the firms' profits are given by

$$\pi_A = P_A \mu_{AA} + 2p_A \mu_{AB}; \qquad \pi_B = P_B \mu_{BB} + 2p_B \mu_{AB}.$$

Given a equilibrium $(p_A^*, P_A^*, p_B^*, P_B^*)$ with the corresponding measures, $\mu_{AA}^*, \mu_{AB}^*, \mu_{BB}^*$ for S_{AA}, S_{AB}, S_{BB}, we say that it is a *mixed bundling equilibrium* if $\mu_{AB}^* > 0$ and that it is a *pure bundling equilibrium* if $\mu_{AB}^* = 0$. It is almost immediate to see that a pure bundling equilibrium exists for any values of parameters as, for each firm, pure bundling is a best response to pure bundling.[35] The next proposition establishes that no mixed bundling equilibrium exists when the dominance of firm A is sufficiently strong. In fact, this result also holds if firm A faces two specialist opponents B1 and B2, that is in each equilibrium firm A plays a pure bundling strategy, such that each consumer either buys firm A's bundle or products B1 and B2, at least as long as we consider symmetric equilibria such that $p_{A1} = p_{A2}$ and $p_{B1} = p_{B2}$. The reason is that when A faces two specialists such that $p_{B1} = p_{B2}$, A's pricing problem coincides with A's problem when A faces a generalist and $P_B = 2p_B$. Hence he has the same incentive to avoid mixed bundling strategies, as we describe immediately after the proposition.

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consumer located at $(s_1, s_2) = (s'', 0)$ (at $(s_1, s_2) = (0, s'')$) is indifferent between the alternatives AA and BA (between the alternatives AA and AB).

[33]The expressions for $\mu_{AA}, \mu_{AB}, \mu_{BB}$ are found in the proof of Proposition 9.

[34]Precisely, if $s' < 0$ then each type of consumer prefers BB to AB (and to BA). If $s'' > 1$, then each type of consumer prefers AA to AB (and to BA).

[35]Let $P_{2,A}^*, P_{2,B}^*$ be the equilibrium prices from Proposition 2. Under mixed bundling, $(p_A^*, P_{2,A}^*, p_B^*, P_{2,B}^*)$ is an equilibrium if p_A^* and p_B^* are large enough, as for firm $A\,(B)$ it is impossible to induce any type of consumer to choose AB or BA since $P_B = P_{2,B}^*$ and a large p_B imply $s' < 0$ for any $p_A \ge 0$, thus $S_{AB} = S_{BA} = \emptyset$ ($P_A = P_{2,A}^*$ and a large p_A imply $s'' > 1$ for any $p_B \ge 0$, thus $S_{AB} = S_{BA} = \emptyset$).

Proposition 9. *Consider the mixed bundling game with $n = 2$. Then both if firm A faces a generalist opponent or two specialists opponents, we have that*
(i) there exists no mixed bundling equilibrium if $f(1) > 0$ and $\alpha \geq t + \frac{t}{f(1)}$;
(ii) when f is the uniform density, there exists no mixed bundling equilibrium if $\alpha \geq \frac{9}{8}t$.

Proposition 9(i) relies on proving that if α is sufficiently large and (p_A, P_A, p_B, P_B) are such that $\mu_{AB} > 0$, then $s'' < 1$ and it is profitable for A to reduce P_A. A small reduction in P_A reduces A's revenue from inframarginal consumers but attracts some marginal consumers. When α is large, the inequality $s'' < 1$ implies that P_A is large. Hence, it follows that the revenue increase (which is proportional to the initial P_A) from the marginal consumers dominates the revenue decrease from inframarginal consumers (which is proportional to the reduction in P_A). This explains why it is profitable to reduce P_A until s'' reaches the value of 1 to make $\mu_{AB} = 0$.[36]

In the case of the uniform distribution, the lower bound on α from Proposition 9(i) is $t + \frac{t}{f(1)} = 2t$, but Proposition 9(ii) relies on some particular features of the uniform distribution to establish that no mixed bundling equilibrium exists if $\alpha \geq \frac{9}{8}t$.[37] In order to see how this stronger result is obtained, fix p_B, P_B arbitrarily and let M_A denote the set of (p_A, P_A) such that $\mu_{AB} > 0$. Whereas Proposition 9(i) is proved by showing that $\frac{\partial \pi_A}{\partial P_A}$ is negative at each $(p_A, P_A) \in M_A$ if $\alpha \geq t + \frac{t}{f(1)} = 2t$, for the uniform distribution we can show that if $\alpha \in [\frac{9}{8}t, 2t)$, there exists no $(p_A, P_A) \in M_A$ such that $\frac{\partial \pi_A}{\partial P_A} = 0$ and $\frac{\partial \pi_A}{\partial p_A} = 0$ are both satisfied; therefore no mixed bundling strategy is optimal for firm A when $\alpha \in [\frac{9}{8}t, 2t)$.

It is interesting to notice that a well-established result in the literature is that mixed bundling reduces profits with respect to independent pricing, at least for symmetric firms: see Armstrong and Vickers (2010) and references therein.[38] Propositions 3(i) and 9(i), conversely, prove that if one firm's dominance over the other is strong enough, that is if $\alpha \geq t + \frac{t}{f(1)}$ and

[36]Proposition 9(i) is linked to a result in Menicucci, Hurkens and Jeon (2015) (MHJ henceforth) about the optimality of pure bundling for a two-product monopolist. See Daskalakis, Deckelbaum and Tzamos (2017) for a similar resut in a monopoly context. In our duopoly setting, given (p_B, P_B) chosen by firm B, the problem of maximizing A's profit with respect to (p_A, P_A) is equivalent to the problem of maximizing the profit of a two-product monopolist facing a consumer with suitably distributed valuations and such that the consumer enjoys a synergy of $2p_B - P_B \geq 0$ if she consumes both objects. Since MHJ do not allow for synergies, strictly speaking Proposition 9(i) is not a corollary of the results in MHJ.

[37]Numeric analysis suggests that (i) no mixed bundling NE exists as long as $\alpha \geq 0.72t$; (ii) when a mixed bundling NE exists, the firms' equilibrium profits are lower than under independent pricing.

[38]Armstrong and Vickers (2010) explain this result by referring to firms' incentives to compete fiercely for

$\alpha > \bar{\alpha}$, then mixed bundling boils down to pure bundling, and each firm's profit is larger under mixed bundling than under independent pricing.

Proof of Proposition 9 (i)

In the case that $0 < s'$ and $s'' < 1$, each of the sets S_{AA}, S_{AB}, S_{BB} has a positive measure as follows:

$$\mu_{AA} = F(s')F(s'') + \int_{s'}^{s''} F(s' + s'' - s_1)f(s_1)ds_1; \qquad \mu_{AB} = F(s')[1 - F(s'')]; \tag{22a}$$

$$\mu_{BB} = [1 - F(s')][1 - F(s'')] + \int_{s'}^{s''} [1 - F(s' + s'' - s_1)]f(s_1)ds_1. \tag{22b}$$

Therefore, given $\pi_A = P_A\mu_{AA} + 2p_A\mu_{AB}$, we find

$$\begin{aligned}\frac{\partial\pi}{\partial P_A} &= \mu_{AA} + P_A[2F(s')f(s'') + \int_{s'}^{s''} F(s' + s'' - s_1)f(s_1)ds_1](-\frac{1}{2t}) - 2p_A F(s')f(s'')(-\frac{1}{2t}) \\ &= F(s')f(s'')\left[\frac{F(s'')}{f(s'')} - \frac{P_A}{t} + \frac{p_A}{t}\right] + \int_{s'}^{s''} f(s_1)f(s' + s'' - s_1)\left[\frac{F(s' + s'' - s_1)}{f(s' + s'' - s_1)} - \frac{P_A}{2t}\right] ds_1\end{aligned}$$

and we prove that $\frac{\partial\pi}{\partial P_A} < 0$, given $s'' < 1$

- First, we prove that $\frac{F(s'')}{f(s'')} - \frac{P_A}{t} + \frac{p_A}{t} < 0$. Since f is log-concave, it follows that $\frac{F}{f}$ is increasing and $\frac{F(s'')}{f(s'')} - \frac{P_A}{t} + \frac{p_A}{t}$ is decreasing in P_A. Since the inequality $s'' < 1$ is equivalent to $p_A + p_B - t + \alpha < P_A$, it follows that $\frac{F(s'')}{f(s'')} - \frac{P_A}{t} + \frac{p_A}{t} < \frac{1}{f(1)} + \frac{t - p_B - \alpha}{t}$, and the latter expression is negative given $\alpha \geq t + \frac{t}{f(1)}$.
- Now we prove that $\frac{F(s'+s''-s_1)}{f(s'+s''-s_1)} - \frac{P_A}{2t} < 0$ for each $s_1 \in [s', s'']$. Since f is log-concave, it follows that $\frac{F(s'+s''-s_1)}{f(s'+s''-s_1)}$ is decreasing in s_1, and at $s_1 = s'$ we obtain the value $\frac{F(s'')}{f(s'')} - \frac{P_A}{2t}$, which is negative since it is smaller than $\frac{F(s'')}{f(s'')} - \frac{P_A}{t} + \frac{p_A}{t} < 0$, given $2p_A \geq P_A$.

Proof of Proposition 9(ii)

Given $b_1 \equiv P_B - p_B + \alpha$, $b_2 \equiv p_B + \alpha \geq b_1$, we say that firm A plays a *pure bundling strategy* if and only if $p_A \geq b_1 + t$ and/or $P_A \leq b_2 - t + p_A$ because $\mu_{AB} = 0$ in either of these cases.[39] Given b_1, b_2, we define M_A as the set of (p_A, P_A) such that $\mu_{AB} > 0$, that is

$$M_A = \{(p_A, P_A) : p_A < b_1 + t, \quad b_2 - t + p_A < P_A \leq 2p_A\}.$$

We say that A plays a *mixed bundling strategy* if $(p_A, P_A) \in M_A$. Notice that M_A is non-empty if and only if $b_1 > -t$ and $b_2 < 2t + b_1$: see Figure 3.

the consumers which choose to buy both products from the same firm. This is closely related to the strong demand elasticity effect we find when $\alpha = 0$, that is when the firms are symmetric.

[39]Precisely, $x' \leq 0$ if and only if $p_A \geq b_1 + t$; $x'' \geq 1$ if and only if $P_A \leq b_2 - t + p_A$.

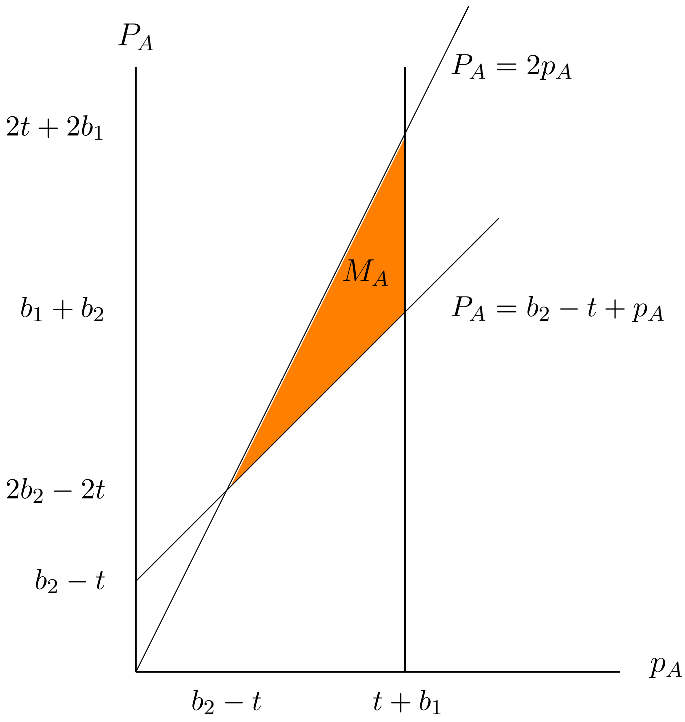

Figure 3: Mixed bundling strategies for firm A.

Using (22), for each $(p_A, P_A) \in M_A$ we have

$$\pi_A = \frac{1}{8t^2}\left(\begin{array}{c} P_A^3 + 4p_A^3 - 2\,(b_1 + b_2 + 2t)\,P_A^2 - 6p_A^2 P_A - 4\,(b_1 - b_2 + 2t)\,p_A^2 + 8\,(b_1 + t)\,P_A p_A \\ +(2t^2 + 4tb_2 + b_2^2 + 2b_1 b_2 - b_1^2)P_A - 4(b_2 - t)(t + b_1)p_A \end{array}\right)$$

and

$$\begin{aligned} \frac{\partial \pi_A}{\partial p_A} &= \frac{1}{8t^2}\left(12p_A^2 - 4\,(3P_A + 4t - 2b_2 + 2b_1)\,p_A + 8\,(b_1 + t)\,P_A - 4(b_2 - t)(t + b_1)\right) \\ \frac{\partial \pi_A}{\partial P_A} &= \frac{1}{8t^2}\left(3P_A^2 - 4\,(2t + b_1 + b_2)\,P_A - 6p_A^2 + 8\,(b_1 + t)\,p_A + 2t^2 + 4tb_2 + b_2^2 + 2b_2 b_1 - b_1^2\right). \end{aligned}$$

Since $\alpha \geq \frac{9}{8}t$ implies $b_1 > \frac{9}{8}t$, we consider the following set $\mathcal{B}$ of possible values for (b_1, b_2): $\mathcal{B} = \{(b_1, b_2) : \frac{9}{8}t < b_1 \leq b_2 < 2t + b_1\}$. We prove that for each $(b_1, b_2) \in \mathcal{B}$ it is never a best reply for firm A to play (p_A, P_A) in M_A, that is the best reply of firm A is a pure bundling strategy. The proof is organized in three steps. In Step 1 we prove that for firm A playing independent pricing (that is, $P_A = 2p_A$) in M_A is suboptimal. A mixed bundling strategy for firm A can thus be optimal only if it lies in the interior of M_A, which implies that the first (and second) order conditions must be satisfied. However, in Step 2 we show that if

$(p_A, P_A) \in M_A$ is such that $\frac{\partial \pi_A}{\partial p_A} = 0$, then P_A must be larger than a suitable $\bar{P}_A$, while in Step 3 we show that $\frac{\partial \pi_A}{\partial P_A} = 0$ implies that P_A must be smaller than $\bar{P}_A$. Hence, it must be optimal for firm A to play a pure bundling strategy whenever $b_2 \geq \frac{9}{8}t$.

Step 1 Suppose that $(b_1, b_2) \in \mathcal{B}$. Playing $(p_A, P_A) \in M_A$ such that $P_A = 2p_A$ is not a best reply for firm A because either $\frac{\partial \pi_A}{\partial p_A} > 0$ and/or $\frac{\partial \pi_A}{\partial P_A} < 0$.
We start by evaluating $\frac{\partial \pi_A}{\partial p_A}$ and $\frac{\partial \pi_A}{\partial P_A}$ at $P_A = 2p_A$ and we find

$$\begin{aligned}
\frac{\partial \pi_A}{\partial p_A} &= \frac{1}{t^2}\left(-\frac{3}{2}p_A^2 + (b_2 + b_1)p_A - \frac{1}{2}(b_2 - t)(t + b_1)\right) \equiv z(p_A), \\
\frac{\partial \pi_A}{\partial P_A} &= \frac{1}{t^2}\left(\frac{3}{4}p_A^2 - (t + b_2)p_A + \frac{1}{8}\left(2b_2 b_1 + b_2^2 + 4tb_2 + 2t^2 - b_1^2\right)\right) \equiv Z(p_A).
\end{aligned}$$

Notice that if $(p_A, P_A) \in M_A$, then $p_A \in (b_2 - t, b_1 + t)$. Let p_A^* denote the larger solution to $z(p_A) = 0$, that is $p_A^* = \frac{1}{3}(b_1 + b_2 + \sqrt{(b_2 - t)^2 + (b_1 + t)(2t + b_1 - b_2)})$, and $b_2 - t < p_A^* < b_1 + t$ since $z(b_2 - t) = \frac{1}{2t^2}(b_2 - t)(b_1 - b_2 + 2t) > 0$ and $z(b_1 + t) = -\frac{1}{2t^2}(b_1 + t)(b_1 - b_2 + 2t) < 0$ in $\mathcal{B}$. In fact, from $z(b_2 - t) > 0 = z(p_A^*)$ we infer that $z(p_A) > 0$ for $p_A \in (b_2 - t, p_A^*)$. This implies that (p_A, P_A) such that $P_A = 2p_A$ and $p_A \in (b_2 - t, p_A^*)$ is not a best reply for A since it is profitable to increase p_A.
For $p_A \in [p_A^*, b_1 + t)$ we prove that $Z(p_A) < 0$. This implies that (p_A, P_A) such that $P_A = 2p_A$ and $p_A \in [p_A^*, b_1 + t)$ is not a best reply for A since it is profitable to reduce P_A. We find $Z(b_1 + t) = -\frac{1}{8t^2}(b_2 - b_1)(2t + b_1 - b_2 + 2t + 4b_1) \leq 0$ in $\mathcal{B}$ and

$$Z(p_A^*) = -\frac{(2t + b_2 - b_1)\left(b_2 + b_1 + 4\sqrt{(b_2 - t)^2 + (b_1 + t)(2t + b_1 - b_2)}\right) - 12t^2}{24t^2}$$

which now we prove to be negative in $\mathcal{B}$. Precisely, we define $\xi_1(b_1, b_2) \equiv (2t + b_2 - b_1)(b_2 + b_1 + 4\sqrt{(b_2 - t)^2 + (b_1 + t)(2t + b_1 - b_2)})$ and show that

$$\xi_1(b_1, b_2) > 12t^2 \qquad \text{for} \quad \text{any} \quad (b_1, b_2) \in \mathcal{B}. \tag{23}$$

To this purpose we prove below that $\frac{\partial \xi_1}{\partial b_2} > 0$ in $\mathcal{B}$, and $\xi_1(b_1, b_1) = 4t(b_1 + 2\sqrt{b_1^2 + 3t^2}) > 12t^2$ for any $b_1 > t$ implies (23). Precisely, $\frac{\partial \xi_1}{\partial b_2} = 2b_2 + 2t + \frac{6b_1^2 + 8b_2^2 - 10b_2 b_1 + 14b_1 t - 10tb_2}{\sqrt{(b_2 - t)^2 + (b_1 + t)(2t + b_1 - b_2)}}$ and $\frac{\partial \xi_1}{\partial b_2} > 0$ in $\mathcal{B}$ since $\xi_2(b_1, b_2) \equiv 6b_1^2 + 8b_2^2 - 10b_2 b_1 + 14b_1 t - 10tb_2 > 0$ in $\mathcal{B}$.[40] ■

Step 2 Suppose that $(b_1, b_2) \in \mathcal{B}$. If $(p_A, P_A) \in M_A$ is such that $\frac{\partial \pi_A}{\partial p_A} = 0$, then $P_A \geq \bar{P}_A$, for a suitable $\bar{P}_A$.

[40] Minimizing ξ_2 over the closure of $\mathcal{B}$ yields the minimum point $b_1 = t$, $b_2 = \frac{5}{4}t$, with $\xi_2(t, \frac{5}{4}t) = \frac{15}{2}t^2 > 0$.

For the equation $\frac{\partial \pi_A}{\partial p_A} = 0$ in the unknown p_A, there exists at least a real solution if and only if $P_A \leq \frac{2}{3}(b_1 + b_2 - \sqrt{(b_1+t)(b_2-t)})$ or $P_A \geq \frac{2}{3}(b_1 + b_2 + \sqrt{(b_1+t)(b_2-t)}) \equiv \bar{P}_A$. We now prove that if (p_A, P_A) is such that $\frac{\partial \pi_A}{\partial p_A} = 0$ and $P_A \leq \frac{2}{3}(b_1 + b_2 - \sqrt{(b_1+t)(b_2-t)})$, then $(p_A, P_A) \notin M_A$; therefore $\frac{\partial \pi_A}{\partial p_A} = 0$ implies $P_A \geq \bar{P}_A$.
First notice that $\frac{2}{3}(b_1+b_2-\sqrt{(b_1+t)(b_2-t)})$ is smaller than b_1+b_2 and in fact it is sometimes smaller than $2b_2 - 2t$ for some $(b_1, b_2) \in \mathcal{B}$. If $\frac{2}{3}(b_1 + b_2 - \sqrt{(b_1+t)(b_2-t)}) > 2b_2 - 2t$, then the line $P_A = \frac{2}{3}(b_1 + b_2 - \sqrt{(b_1+t)(b_2-t)})$ has a non-empty intersection with M_A, and we find that (i) at $p_A = P_A - b_2 + t$ (i.e., along the south-east boundary of M_A) $\frac{\partial \pi_A}{\partial p_A} = \frac{1}{2}(b_2 - t)(b_1 + b_2 - P_A)$, which is positive given $P_A \leq \frac{2}{3}(b_1 + b_2 - \sqrt{(b_1+t)(b_2-t)})$; (ii) $\frac{\partial \pi_A}{\partial p_A}$ is decreasing with respect to p_A for $p_A \leq \frac{1}{2}P_A + \frac{1}{3}(b_1 - b_2) + \frac{2}{3}t$, and $P_A - b_2 + t < \frac{1}{2}P_A + \frac{1}{3}(b_1 - b_2) + \frac{2}{3}t$ given $P_A \leq \frac{2}{3}(b_1 + b_2 - \sqrt{(b_1+t)(b_2-t)})$. Therefore $\frac{\partial \pi_A}{\partial p_A} > 0$ for each $(p_A, P_A) \in M_A$ such that $P_A \leq \frac{2}{3}(b_1 + b_2 - \sqrt{(b_1+t)(b_2-t)})$, and in fact for each $(p_A, P_A) \in M_A$ such that $P_A < \bar{P}_A$. $\blacksquare$

Step 3 Suppose that $(b_1, b_2) \in \mathcal{B}$ and that $b_2 \geq \frac{9}{8}t$. If $(p_A, P_A) \in M_A$ is a best reply for firm A, then $P_A < \bar{P}_A$.
The equation $\frac{\partial \pi_A}{\partial P_A} = 0$ is quadratic and convex in P_A. In order to satisfy the second order condition, the best reply for firm A must be such that P_A is equal to the smaller solution of $\frac{\partial \pi_A}{\partial P_A} = 0$. We now show that $\frac{\partial \pi_A}{\partial P_A} < 0$ at $P_A = \bar{P}_A$, which implies that the smaller solution to $\frac{\partial \pi_A}{\partial P_A} = 0$ is smaller than $\bar{P}_A$. We find

$$\frac{\partial \pi_A}{\partial P_A} = -\frac{3}{4t^2}p_A^2 + \frac{b_1+t}{t^2}p_A + \frac{2b_2b_1 - 7b_1^2 - b_2^2 - 20tb_1 + 2t^2 - 16t\sqrt{(b_2-t)(b_1+t)}}{24t^2} \equiv W(p_A)$$

and notice that $\bar{P}_A < b_1 + b_2$; therefore W is defined for $p_A \in (\frac{1}{2}\bar{P}_A, \bar{P}_A - b_2 + t)$. We prove that $W(p_A) < 0$ for each $p_A \in (\frac{1}{2}\bar{P}_A, \bar{P}_A - b_2 + t)$, and to this purpose we notice that W is maximized with respect to p_A at $p_A = \begin{cases} \frac{2}{3}t + \frac{2}{3}b_1 & \text{if } b_2 \leq \frac{3-\sqrt{5}}{2}b_1 + \frac{5-\sqrt{5}}{2}t \\ \frac{1}{2}\bar{P}_A & \text{if } b_2 > \frac{3-\sqrt{5}}{2}b_1 + \frac{5-\sqrt{5}}{2}t \end{cases}$.

- If $b_2 \leq \frac{3-\sqrt{5}}{2}b_1 + \frac{5-\sqrt{5}}{2}t$, then $b_1 \leq \sqrt{5}t$ in order to satisfy $b_1 \leq b_2$, and $W(\frac{2}{3}t + \frac{2}{3}b_1) = \frac{1}{12t^2}(5t^2 - 2b_1t - \frac{1}{2}b_2^2 + b_2b_1 + \frac{1}{2}b_1^2 - 8t\sqrt{(b_1+t)(b_2-t)}) \equiv \xi_3(b_1, b_2)$, which is decreasing in b_2 and $\xi_3(b_1, b_1) = \frac{1}{12t^2}(5t^2 - 2tb_1 + b_1^2 - 8t\sqrt{b_1^2 - t^2})$ is negative for $b_1 \in [\frac{9}{8}t, \sqrt{5}t]$.

- If $b_2 > \frac{3-\sqrt{5}}{2}b_1 + \frac{5-\sqrt{5}}{2}t$, then we evaluate $W(\frac{1}{2}\bar{P}_A) = \frac{1}{24t^2}(4t^2 - 10tb_1 + 6tb_2 - b_1^2 - 3b_2^2 + 4b_1b_2 - 4(2t - b_1 + b_2)\sqrt{(b_1+t)(b_2-t)})$, and we prove it is negative. Precisely, we show that

$$\xi_4(b_1, b_2) \equiv 4\,(2t - b_1 + b_2)\sqrt{(b_2 - t)\,(b_1 + t)} - 4t^2 + 10tb_1 - 6tb_2 + b_1^2 + 3b_2^2 - 4b_1b_2$$

is positive, and from $b_1 + t > b_2 - t$ we obtain $\xi_4(b_1, b_2) > 4\,(2t - b_1 + b_2)\,(b_2 - t) - 4t^2 + 10tb_1 - 6tb_2 + b_1^2 + 3b_2^2 - 4b_1b_2 = b_1^2 + 7b_2^2 - 8b_1b_2 - 12t^2 + 14tb_1 - 2tb_2 \equiv \xi_5(b_1, b_2)$. It is immediate that ξ_5 is increasing with respect to b_2, and $\xi_5(b_1, \frac{3-\sqrt{5}}{2}b_1 + \frac{5-\sqrt{5}}{2}t) = -\frac{1}{2}(13\sqrt{5} - 27)b_1^2 + (61 - 23\sqrt{5})tb_1 - \frac{1}{2}(33\sqrt{5} - 71)t^2 > 0$ for $b_1 \in (\frac{9}{8}t, \sqrt{5}t)$; $\xi_5(b_1, b_1) = 12t\,(b_1 - t) > 0$. ■

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