Int J Game Theory (2003) ██:███–███

International Journal of Game Theory

Evolutionary insights on the willingness to communicate*

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Received: September 2000/Revised: March 2003

Abstract. While in previous models of pre-play communication players are forced to communicate, we investigate what happens if players can choose not to participate in this cheap talk. Outcomes are predicted by analyzing evolutionary stability in a population of a priori identical players. If the game following the communication rewards players who choose the same action then an efficient outcome is only guaranteed when participation in the pre-play communication is voluntary. If however players aim to coordinate on choosing different actions in the underlying game and there are sufficiently many messages then the highest payoff is selected when players are forced to talk to each other before playing the game.

Journal of Economic Literature Classification Numbers: C72, C79.

Key words: evolutionarily stable sets, pure coordination, cheap talk.

1. Introduction

Nash equilibrium outcomes are often inefficient. Even when Pareto superior Nash equilibrium outcomes exist, such inefficient outcomes are difficult to rule out when they are associated with strict equilibria. Play is easily locked in as each player is doing the best he can do given his correct beliefs of what others will do. Informally it is often argued that players will nonetheless coordinate

* S. Hurkens gratefully acknowledges financial support from CIRIT, Generalitat de Catalunya, Grant 1997SGR 00138 and Grant BEC2000-1029. K. Schlag gratefully acknowledges financial support from the Deutsche Forschungsgemeinschaft, Sonderforschungsbereich 303 at the University of Bonn. We thank the associate editor and two anonymous referees for extremely helpful comments.

on achieving the best outcome if this is supported by an equilibrium which dominates all other outcomes. One justification for this is that in many real life applications players will communicate with their opponents before actually playing the game. During this communication they should then reach consensus to coordinate on the efficient outcome.

However, the formal introduction of pre-play communication does not eliminate any Nash equilibrium outcome, as it is always an equilibrium to speak randomly and ignore messages and play any equilibrium of the underlying game. In fact, pre-play communication extends the set of Nash equilibria since players may, for example, deliberately speak and actually listen and react to messages in order to coordinate actions on one of two inefficient equilibria of the underlying game. In recent years it has been argued that only the efficient outcomes are evolutionarily stable. For example, Swinkels (1992), Matsui (1992), Sobel (1993), and Kim and Sobel (1995) show that various notions of evolutionary stability (EES set, CSS, NES, and stochastically stable set, respectively) yield efficient outcomes in two person coordination games[1] with pre-play communication. All these notions of evolutionary stability are based on the assumption that players can be divided into two types or populations. This implies that behavior in the two populations may be distinct and this is crucial for the result that inefficient Nash equilibria are not evolutionarily stable, even if such an equilibrium is symmetric.[2]

The two population assumption may be appropriate in case of men and women trying to coordinate on greetings (kisses or handshakes), buyers and sellers settling on a price, or employers and employees fixing a wage. On the other hand, this asymmetry assumption may not be adequate in the case of software agents negotiating bandwith on the internet (Vulkan, 2001), traders setting bid and ask prices in a double auction, or employees determining wage demands. Moreover, even if several external asymmetries exist (e.g. sex, age, and eye color), role identification may in fact be very noisy and it becomes evolutionarily stable to ignore external signals all together, returning us to a single population model (see Binmore and Samuelson, 2001). Furthermore, traditionally evolutionary models and solution concepts have been considered for symmetric games where players are drawn from a single population. Examples are Maynard Smith and Price's (1973) Evolutionarily Stable Strategy, Maynard Smith's (1982) Neutrally Stable Strategy, and Thomas' (1985) Evolutionarily Stable set. In this paper we focus on single population models.

It turns out that inefficient outcomes of coordination games with pre-play communication can be supported by evolutionarily stable strategies in the single population model (see Wärneryd (1991, 1998), Schlag (1993, 1994)), and Section 3 for an example). The possible emergence of inefficient outcomes and the common belief that typically the efficient outcome should result in such simple games even if players are symmetric leads us to believe that some features of communication have not been modelled adequately.

[1] A pure coordination game is a symmetric simultaneous move game in which each player has a finite set of pure strategies; play of the same strategy results in a strictly positive payoff, miscoordination leads to a payoff of 0.

[2] Consider a symmetric and inefficient Nash equilibrium in which all messages are used. First drift may take population 1 to a state where some message m is not sent. Then drift may take population 2 to a state were the efficient action would be played after receiving m. Finally, drift may take population 1 to a state where m is sent and the efficient action is played.

In the literature cited above, pre-play communication is modelled by assuming that players meet once before the game to simultaneously send the other a message where messages have no literal meaning. Choice of action in the later play of the actual game can then be conditioned on the message sent and received during the communication round. Notice that communication is mandatory in the sense that players are forced to send a message and to listen to the message sent by the opponent. (Of course, they may choose not to condition play on messages received.) The new idea of this paper is to relax this assumption and to give a player the opportunity not to show up to the round of pre-play communication. We thereby assume that a player who does not show up to pre-play does not learn whether his opponent went to the pre-play communication, in particular he does not receive any messages sent by his opponent. Thus, not to show up is an irreversible commitment to not listen to sent messages.

The ability to commit by avoiding the communication round has drastic effects on the evolutionary stability of outcomes in the case where the underlying game is one of coordination. The efficient outcome becomes the unique evolutionarily stable outcome.[3,4]

Our above result reveals the importance of making pre-play communication voluntary in *coordination games*. In the second half of our paper we investigate whether voluntary communication is also effective in games where players want to choose different actions. For this we choose as the underlying game a *task allocation game*; the choice of the same action leads to payoff zero while choice of different actions leads to strictly positive payoffs. This makes it much like a coordination game except that an efficient outcome cannot be reached by a symmetric strategy profile. We add a conflict of interest and assume that payoffs are not identical when players choose different actions which causes there to be more than one efficient outcome.

In contrast to coordination games, efficient outcomes are only reached by asymmetric strategy profiles which means that an efficient outcome cannot be reached in the single population model. However, communication gives players the opportunity to behave asymmetrically and we find under either mandatory or voluntary communication that all evolutionarily stable outcomes induce higher payoffs than when communication is not possible. The level of the induced payoff depends on how players coordinate their actions after messages have been sent. Induced payoffs are far from the efficient when players sometimes disregard the outcome of pre-play communication. In particular, all ESS are far from being efficient when communication is voluntary as players sometimes choose the option not to show up to pre-play communication. A high degree of coordination among the players is only possible when communication is mandatory. Here we find nearly efficient ESS when there are many messages and each player performs each task approximately equally often after

[3] We consider Evolutionarily Stable Strategies and Evolutionary Stable Sets as evolutionarily stable outcomes. The notion of Neutral Stable Strategy would not exclude all inefficient outcomes. (See section 4 in Wärneryd, 1991).

[4] Van Damme and Hurkens (1996) already showed how efficiency can be guaranteed in coordination games if commitment opportunities are introduced. In their model, players choose when to irreversibly commit to playing an action with previous commitments of others being observable. While in their model the way in which payoffs in the game are achieved is changed, commitment is more subdued in our model.

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each message. Thus, contrasting to coordination games, we find voluntary communication less appealing in task allocation games.

Interestingly in the two population setting, while evolution yields efficiency (being a strict equilibrium) without communication, the possibility to communicate either destroys stability (Schlag, 1994) or enlarges the set of predictions to include inefficient outcomes (Kim and Sobel, 1995).[5]

The rest of the paper is organized as follows. In Section 2 the basic concepts are introduced, then in Section 3 results on coordination games are presented. Section 4 contains our analysis of task allocation games with both mandatory and voluntary pre-play communication. Section 5 contains the conclusion.

2. Evolutionary stability and pre-play communication

In this paper we consider a specific two person game in which we select strategies and outcomes according to evolutionary stability.

Consider a symmetric finite two person normal form game in which players have N pure strategies $\{1, \ldots, N\}$ and A is the $N \times N$ payoff matrix. Let Δ_{N-1} denote the set of mixed strategies described as probability distributions on the set of pure strategies. Then the payoff of strategy x when meeting y equals $x \cdot Ay$ for $x, y \in \Delta_{N-1}$. x is called a *Nash strategy* if (x, x) is a Nash equilibrium, i.e., if $x \cdot Ax \geq y \cdot Ax$ for all strategies y. We say that x *induces (an expected payoff)* $x \cdot Ax$, $\max x \cdot Ax$ is called the *efficient payoff*.

Evolutionary stability is formulated for the following environment. Two players are drawn at random from a large (essentially, infinite) population of identical individuals to play a symmetric game. A Nash strategy x is an *Evolutionarily Stable Strategy (ESS)* (Maynard Smith and Price, 1973) if for every strategy $y \neq x$, $y \cdot Ax = x \cdot Ax$ implies that $y \cdot Ay < x \cdot Ay$. Note that if (x, x) is a quasi-strict Nash equilibrium (*i.e.* all pure best replies against x are used with positive probability in x) for every strategy $y \neq x$ with $y \cdot Ax = x \cdot Ax$ the condition $y \cdot Ay < x \cdot Ay$ is equivalent to $(y - x) \cdot A(y - x) < 0$. It follows (see Van Damme (1987, Thm. 9.2.7)) that a quasi-strict Nash strategy x is an ESS if and only if A is negative definite with respect to the set of pure best replies against x, $B(x)$, that is, if and only if

$$z \cdot Az < 0 \text{ for all } z \in \Re^N \text{ with } z \neq 0, \quad \sum z_i = 0, \text{ and } z_i = 0 \text{ if } i \notin B(x).$$

In some games ESS do not exist and we therefore also consider a setwise stability concept. A subset X of the set of all Nash strategies is an *Evolutionarily Stable Set* (ES Set, Thomas, 1985) if it is nonempty and for each $x \in X$ and each y, $y \cdot Ax = x \cdot Ax$ implies that either

(i) $y \cdot Ay < x \cdot Ay$, or
(ii) $y \cdot Ay = x \cdot Ay$ and $y \in X$.

[5] While this result is easily verified formally, Kim and Sobel (1995) only hint to this weakness in their discussion (see paragraph following Proposition 3 on page 1189).

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Any singleton ES Set contains an ESS and every ESS constitutes an ES Set as a singleton, so that ES Sets can be seen as the set-valued extension of the ESS concept.

In our analysis we will consider evolutionary stability in specific games that involve mandatory or voluntary communication. We will consider a symmetric game with strategy set K (such as a coordination game or a task allocation game) that we refer to as the base game and denote by $\mathcal{G}$. Then we extend the game $\mathcal{G}$ by adding a round of pre-play communication, also called cheap talk, before the players play the coordination game.

In the case of mandatory communication the two players simultaneously send each other a message from a given finite message set $M = \{m_1, \dots, m_n\}$ with $n \geq 2$. Sending and receiving messages is costless and only affects later play in the game as players can condition behavior on the message they receive. A pure strategy (of the reduced normal form game) now consists of a pair (m, f) where m is the message he sends and $f : M \to K$ is a decision rule that specifies to play action $f(m_i)$ after receiving message m_i. This induces a symmetric finite game $\mathcal{G}^M$ whose associated payoff matrix we denote by C. Notice that the payoff a pure strategy (m, f) receives when playing against pure strategy (m', f') is $e_{f(m')} \cdot Ae_{f'(m)}$.

In the case of voluntary communication we now add the possibility that a player can choose not to show up to the pre-play communication. It will be assumed that after such a choice, the player cannot observe whether his opponent was willing to communicate or not. Formally, this is modelled by adding an additional message m^H to M (so now $M^V = \{m^H, m_1, \dots, m_n\}$) and adjusting the decision rule as follows. A pure strategy for a player is a pair (m, f) as before with the restriction that $m = m^H$ implies $f(\tilde{m}) = f(m')$ for all $\tilde{m}, m' \in M^V$. We will also write (m^H, k) instead of (m^H, f) where $f(m) = k$ for all $m \in M^V$. The payoff matrix of the induced game will be denoted by W. Notice that the payoff a pure strategy (m, f) receives when playing against pure strategy (m', f') is $e_{f(m')} \cdot Ae_{f'(m)}$.

3. Communication before a coordination game

Assume that the base game $\mathcal{G}_1$ is a generic *coordination game* defined as follows. Players have action set $K = \{1, \dots, k\}$. Whenever the two players choose different actions then both players would be better off if exactly one of the two players chooses the action used by the other player, i.e., $a_{ii} > \max\{a_{ij}, a_{ji}\}$ for all $j \neq i$. Index actions such that $a_{11} > a_{22} > \cdots > a_{kk}$. The special case in which $a_{ij} = 0$ for $i \neq j$ is called a *pure coordination game*. All pure strategies are evolutionarily stable in the base game.

3.1. Mandatory communication

Mandatory communication has been analyzed in the literature before. In this subsection we collect the established results for mandatory communication in coordination games.

It is easily shown that the set of all strategies that induce the efficient payoff a_{11} when played against themselves is an ES Set. In each matching only action 1 is played although it can be that the opponent would play a different action if he had received a different message. Moreover, there is no other ES Set in which the same action is played in all matchings (see Schlag, 1993, 1994). However, as pointed out by Schlag (1993, 1994) and Wärneryd (1998), cheap talk does not guarantee efficiency as there exist ESS with payoff strictly less than a_{11}. Fix i and j with $i < j$ and $i, j \in K$. Consider the mixed strategy x^{ij} where a player mixes uniformly over all messages and plays action j if the messages are the same and action i otherwise. Then x^{ij} is a Nash strategy that puts positive weight on each of its best replies and x^{ij} earns $b = (a_{jj} + (n-1)a_{ii})/n < a_{ii}$ against itself. Suppose that x' is a best reply to x^{ij} that uses a non-uniform probability distribution over the messages. Then x' earns less than b when meeting itself because the probability of two identical messages is more than $1/n$. Thus, x^{ij} is an ESS that achieves payoffs bounded below a_{ii} irrespective of the number of messages.

3.2. Voluntary communication

We will now consider the possibility that a player can choose not to show up to the pre-play communication. We will show that voluntary pre-play communication induces efficiency:

Proposition 1. *Let $M^V = \{m^H, m_1, \ldots, m_n\}$ and let $\mathscr{G}_1$ be a coordination game. Then $\mathscr{G}_1^{M^V}$ has a unique ES Set, namely $X = \{x : x \cdot Wx = a_{11}\}$.*

Proof: Note that X is the set of mixed strategies that induce the efficient payoff. X is the set of strategies where a player either does not show up to pre-play communication and plays action 1 in the game or randomizes between some cheap talk messages and plays action 1 whenever he receives one of the messages that were sent with positive probability.

We first verify that if for some $x \in X$, x is contained in some ES Set X', then $X \subset X'$. Let $m \in M^V$ be used with positive probability in x. Then the pure strategy x' that sends m and plays action 1 (independent of the message received) is a best reply against x. Since $x' \cdot Wx' = a_{11} = x \cdot Wx'$, we have $x' \in X'$. Let $x'' \in X$ and suppose it uses m with positive probability. Then it must play action 1 after receiving message m, so $x'' \cdot Wx' = a_{11} = x' \cdot Wx''$ and it follows that $x'' \in X'$. Let $\tilde{x} \in X$ and suppose it does not send m but that it chooses action 1 whenever it receives m. Then again $\tilde{x} \cdot Wx' = a_{11} = x' \cdot W\tilde{x}$ and $\tilde{x} \in X'$. Finally, let $\tilde{y} \in X$ behave just like $\tilde{x}$ except that it does not always choose action 1 after receiving m. Then $\tilde{y}$ is a best reply to $\tilde{x}$ and $\tilde{x} \cdot W\tilde{y} = a_{11} = \tilde{y} \cdot W\tilde{y}$ and $\tilde{y} \in X'$.

Next we show that X is indeed an ES Set. Suppose mutant y is a best reply against a strategy $x \in X$ but $y \notin X$. We need to show that $y \cdot Wy < x \cdot Wy$. The presumptions about the mutant imply that $y \cdot Wx = x \cdot Wx = a_{11}$ and $y \cdot Wy < a_{11}$. Since the game is a coordination game in which the highest payoff is a_{11}, $y \cdot Wx = a_{11}$ implies that also $x \cdot Wy = a_{11}$. Hence, it follows that $y \cdot Wy < a_{11} = x \cdot Wx = y \cdot Wx = x \cdot Wy$.

It remains to be shown that no strategy outside X can be contained in any ES Set. Let $X' \neq X$ be an alternative ES Set. Let $x \in X'$ be a strategy that induces a payoff below the maximal payoff a_{11} against itself, *i.e.* $x \notin X$. Suppose first that for some $i > 1$, (m^H, i) is used with positive probability in x. Since (m^H, i) yields a_{ii} against itself and at most a_{ii} against any other strategy

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(including x), we must have that $(m^H, i) \in X'$. Then $y = (m_1, f)$ with $f(m^H) = f(m_1) = i$ and $f(m_2) = 1$ is a best response against (m^H, i) that yields the same payoff a_{ii} against itself. Hence, $y \in X'$. However, y is not a best reply against itself, that is, y is not a Nash strategy. This yields a contradiction. Hence, (m^H, i) with $i > 1$ is not in the support of $x \in X'$. This implies that $(m^H, 1)$ is not in the support of x either: If it were, any pure communication strategy (m_j, f') in the support of x must have $f'(m^H) = 1$, which would imply that x yields a_{11} against itself, a contradiction. Thus, m^H is not sent in x and there is no evolutionary selection pressure against strategies in the support of x reacting to m^H by playing action 1. This implies that the strategy $\tilde{x}$ that behaves just like x except that it reacts to m^H with action 1, is an element of X. However, this strategy $\tilde{x}$ is not even a Nash strategy, since $(m^H, 1)$ does strictly better against it. □

The efficiency result for voluntary communication stands in stark contrast with the existence of inefficient ESS in the case of mandatory communication. In models of pre-play communication, efficiency can be guaranteed if the population can always drift to a state in which some message is not sent. Following our previous discussion of mandatory communication, all messages are sent in the inefficient ESS and drift is not possible as each pure strategy in the support does better against all others in the support than against itself. Under voluntary communication, not showing up to communication is like a message so inefficiency can only arise if the player does not show up with positive probability. However, here drift is possible as not showing up followed by any action does weakly better against itself than against any other pure strategy.

4. Communication before a 2 × 2 task allocation game

In this Section we aim to gain some understanding of play in games in which players want to choose different actions. For simplicity we will focus on a 2×2 game $\mathcal{G}_2$ with the following payoff matrix

$$A = \begin{bmatrix} 0 & b \\ 1 & 0 \end{bmatrix}$$

where $b > 1$.[6]

We call $\mathcal{G}_2$ a *task allocation game* since players want to choose different actions or tasks. For example, two tasks need to be accomplished and it does not matter who performs which task as long as both tasks are completed. $b > 1$ may indicate that task 1 is more pleasant conditional on the two players coordinating on different tasks. Alternatively, consider two individuals who are walking side by side come to a door that needs to be opened and only allows for one to pass at a time. Here action 2 could describe opening the door and passing through second. Or two cars could simultaneously arrive at a four

[6] While the expressions for the explicit equlibrium strategies and values are more intricate, the general results also go through for any 2×2 game with payoff matrix A such that $a_{12} > a_{21} > a_{11} = a_{22}$. We conjecture that it would go through as well for any 2×2 game with payoff matrix A such that $a_{12} > a_{21} > \max\{a_{11}, a_{22}\}$.

way stop and action 1 (2) could describe to drive first (second). Or bandwith can be divided into a large and a small chunk and action 1 could describe to demand a large chunk.

$\mathcal{G}_2$ has the same best reply structure as a Hawk-Dove Game. When there is no pre-play communication then this game has a unique ESS $z^* := \left(\frac{b}{1+b}, \frac{1}{1+b}\right)$ and we find $z^* \cdot Az^* = \frac{b}{1+b} < 1$.

4.1. Mandatory communication before task allocation

Consider mandatory pre-play communication as defined in Section 2. Again C denotes the payoff matrix of the enlarged game. When the number of messages is n, the efficient payoff is $\frac{1}{n}\frac{1+b}{4} + \frac{n-1}{n}\frac{1+b}{2}$. This payoff is, for example, obtained by the strategy $\hat{x}$ that sends all messages with equal probability, plays action 1 (2) if the received message has lower (higher) index than the sent message, and plays both actions with equal probability if sent and received message coincide. Clearly, this strategy is not an ESS as it is not even a Nash strategy. First we derive some implications of evolutionary stability and present these first verbally along with some intuition.

Consider an element x of an ES Set of the task allocation with mandatory communication. All messages must be used in x as otherwise there are no counterforces to prevent arbitrary drift in behavior to an unsent message. In particular, drift could yield a situation in which sending any unsent message is rewarded with the highest payoff b in the underlying game. But this contradicts the fact that elements of an ES Set are Nash strategies. Since (x, x) is a Nash equilibrium it must prescribe a Nash equilibrium of the task allocation game after each combination of sent and received messages. When sent and received message coincide then symmetry implies the play of the ESS z^* of the underlying game. An asymmetric situation arises when sent and received message do not coincide. In such situations players will coordinate on off diagonal outcomes as it turns out that playing z^* is not evolutionarily stable in this case.[7] It follows that all elements of an ES Set are quasi-strict Nash strategies and ESS themselves. While induced payoffs remain higher than when communication is not possible, they are bounded above by 1 if n and b are small. Similarly, the induced payoff will be inefficient if the strategy puts positive weight on a strategy that chooses the same action after any received message.

Lemma 2. *Let $M = \{m_1, \ldots, m_n\}$ and let $\mathcal{G}_2$ be a task allocation game.*

(i) *Each element x of an ES Set of $\mathcal{G}_2^M$ is an ESS, has each of its best replies in its support and induces an expected payoff in $\left(\frac{b}{1+b}, \frac{1}{n}\frac{b}{1+b} + \left(1 - \frac{1}{n}\right)\frac{1+b}{2}\right]$. In particular, if $n < \frac{b^2+1}{b^2-1}$ then the expected payoff induced by any ESS will lie below 1. If the player using x sometimes chooses action k regardless of what happened during communication then the induced payoff lies below $\frac{2b}{1+b}$ if $k = 1$ and below 1 if $k = 2$.*

(ii) *Consider two players matched with each other each playing the same ESS x of $\mathcal{G}_2^M$. Then each message is sent with positive probability, when two different*

[7] These coordination results are very much in the spirit of Selten (1980) who considers players conditioning on exogenous signals.

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messages are sent then the players coordinate on an off diagonal outcome while they play the ESS z^* of the underlying game when they both send the same message.

Proof: Consider an element x of an ES Set. We first show that when two different messages are sent then the players coordinate on an off-diagonal outcome while they play z^* conditional on sending the same message. Let λ_i be the probability that message m_i is sent. Let x_{ij} be the mixed action played in the task allocation game conditional on sending message m_i and receiving message m_j. Evaluating the ES Set conditions at x against deviations from x_{ij} for given $i, j \in \{1, \dots, n\}$ we obtain the following. If $i = j$ and $\lambda_i > 0$ then x_{ii} must be a Nash strategy in the base game $\mathcal{G}_2$ and thus, $x_{ii} = z^*$ whenever $\lambda_i > 0$. Assume $i \neq j$ and $\lambda_i \lambda_j > 0$. Then (x_{ij}, x_{ji}) must be a Nash equilibrium of $\mathcal{G}_2$. Suppose that $(x_{ij}, x_{ji}) = (z^*, z^*)$. Let x' behave just like x except for the fact that it plays action 1 after sending m_i and receiving m_j. Then $x' \cdot Cx = x \cdot Cx$ while $x' \cdot Cx' = x \cdot Cx'$ so that x' must be part of the ES Set. However, x' is not even a Nash strategy. Thus, $(x_{ij}, x_{ji}) \in \{((1,0),(0,1)),((0,1),(1,0))\}$. In particular, induced payoffs are greater than $b/(1+b)$. Notice that the probability that sent and received message coincide equals $\sum \lambda_i^2 \geq 1/n$ and that the expected payoff conditional on sending different messages equals $(1+b)/2$. Thus, induced payoffs are below $\frac{1}{n}\frac{b}{1+b} + \left(1 - \frac{1}{n}\right)\frac{1+b}{2}$. Finally assume that $\lambda_i = 0$. For any j, x_{ji} does not influence $x \cdot Cx$ and thus there is an element x' of the ES Set such that x' behaves like x except that $x'_{ji} = 2$ for all j. However, x' is then not a Nash strategy as $(m_i, f \equiv 1)$ is a best response to x' as it yields $b > (1+b)/2 > x' \cdot Cx'$. Thus all messages are sent with strictly positive probability in x.

Next we show that x is an ESS. Since the above shows that x is a quasi-strict Nash strategy, x is an ESS if and only if C is negative definite with respect to the support of x. Since in x all messages are used with positive probability, because both actions are played with positive probability when sent and received message coincide ($x_{ii} = z^*$) and since $x_{ij} = 1 - x_{ji} \in \{0, 1\}$ when $i \neq j$, there are $2n$ pure strategies in the support of x. Let B be the $2n \times 2n$ payoff matrix resulting from the restriction of C to the pure strategies in the support of x. Then B is a matrix with 0 on the diagonal and with b or 1 off the diagonal such that $B_{ij} + B_{ji} = 1 + b$ for all $i \neq j$. Let $z \in \Re^{2n} \backslash \{0\}$ with $\sum z_i = 0$. Note that

$$z \cdot Bz = \frac{(1+b)}{2} \sum_i \sum_{j \neq i} z_i z_j = -\frac{(1+b)}{2} \sum_i z_i^2 < 0.$$

It then follows that C is negative definite with respect to the support of x, so that x is in fact an ESS.

Now consider an ESS x that contains (m_i, k) in its support. If $k = 2$ then $x \cdot Cx = e_{(m_i,2)} \cdot Cx$ implies $x \cdot Cx < 1$. Now consider $k = 1$. Let y be the strategy defined as x conditional on not playing $(m_i, 1)$. Let μ be the probability that x puts on playing $(m_i, 1)$. Let $z_1 = e_{(m_i,1)} \cdot Cy$; $z_2 = y \cdot Ce_{(m_i,1)}$; $z_3 = y \cdot Cy$. Then $x \cdot Cx = e_{(m_i,1)} \cdot Cx = (1-\mu)z_1$ and $x \cdot Cx = y \cdot Cx = \mu z_2 + (1-\mu)z_3$. It follows that $\mu = \frac{z_1 - z_3}{z_1 + z_2 - z_3}$. We also know that $0 < z_2 \leq 1$, $0 < z_3 < (b+1)/2$, $z_1 > z_3$, $z_1 + z_2 \leq b + 1$. Maximization of $x \cdot Cx = \frac{z_1 z_2}{z_1 + z_2 - z_3}$ under these constraints implies $x \cdot Cx \leq 2b/(1+b)$. □

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Next we present some insights regarding existence of ESS's in task allocation games under mandatory pre-play communication. First we construct an inefficient ESS that induces an expected payoff below 1. This strategy satisfies the conditions of the lemma above, plays action one (two) whenever received message has lower (resp. higher) index than the sent message and achieves the same expected payoff with each message. Notice that, independently of the number of messages, our candidate inefficient ESS induces an expected payoff below 1 by Lemma 2 (i) as one of its strategies sends message m_1 and always plays action two.[8] We also construct an ESS that induces an expected payoff close to efficiency provided the number of messages is large. Each pure strategy in its support selects action one and action two with nearly the same probability.

Proposition 3. *Let* $M = \{m_1, \ldots, m_n\}$ *and let* $\mathcal{G}_2$ *be a task allocation game. Then*

(i) $\mathcal{G}_2^M$ *has an ESS* x *that induces an expected payoff strictly below 1.*

(ii) *For every* $\delta > 0$ *there exists* n_0 *such that if* $n > n_0$ *then there exists an ESS of* $\mathcal{G}_2^M$ *that induces an expected payoff above* $\frac{1+b}{2} - \delta$.

Proof:

Part (i). Consider the following strategy x: Send message m_j with probability $r_j = b^{2(j-1)}/(1 + b^2 + b^4 + \cdots + b^{2n-2})$. When the message received is the same as the one sent, play the mixed equilibrium of the base game $\left(\frac{b}{1+b}, \frac{1}{1+b}\right)$. If the message received has a higher index than the message sent, play action 2. If the message received has a lower index than the one sent, play action 1. Thus, x puts positive probability on the pure strategy e to send message m_n and then to always play action 1 and on the pure strategy $\tilde{e}$ to send message m_1 and then to always play action 2. It is easily verified that x is a Nash strategy. For a given message received, players prefer to send a message with a higher index. On the other hand, the higher the index of the own message, the higher the probability that the same messages are sent. To make players indifferent between the messages we need the probabilities r_j described above. This strategy earns (against itself) $q := x \cdot Cx = 1 - 1/(1 + b + \cdots + b^{2n-1})$. It is clear that x is quasi-strict. Hence, x is an ESS if and only if C is negative definite with respect to set of pure strategies in the support of x. Let us rename the pure strategies $e_1, \ldots, e_{2n}$ in the support of x as follows: e_{2j-1} denotes the pure strategy to send message m_j and to play action 1 if the message received has index $i \le j$, and to play action 2 if the message received has an index $i > j$. Similarly, e_{2j} denotes the pure strategy to send message m_j and to play action 1 if the message received has index $i < j$, and to play action 2 if the message received has an index $i \ge j$. Let B denote the $2n \times 2n$ matrix obtained from C by restricting it to these strategies. Then $B_{kk} = 0$, $B_{kl} = b$ when $k < l$ and $B_{kl} = 1$ when $k > l$. It is straightforward to check that

$$z \cdot Bz = -\frac{(1+b)}{2} \sum_i z_i^2 < 0$$

[8] Note that this inefficient ESS uses the same pure strategies as the efficient strategy $\hat{x}$ does. The difference lies of course within the probabilities with which these pure strategies are used.

for all $z \in \Re^{2n} \backslash \{0\}$ with $\sum_i z_i = 0$. This proves that C is negative definite with respect to the support of x. Hence, x is an ESS.

Part (ii). The actual construction of a near efficient ESS depends on whether the number of messages is odd or even. We first give a near efficient ESS in the case of an odd number of messages, as the construction is easier.

Let $n = 2k + 1$. Consider the strategy x' that mixes with equal probability over all messages, playing the ESS of the base game in case of equal messages, playing action 2 if the index of the message received j is in the set $\{i+1, \dots, i+k\}$ (mod n) where i is the index of the message sent and playing action 1 otherwise. Notice that $x' \cdot Cx' = \left(1 - \frac{1}{n}\right)\frac{1+b}{2} + \frac{1}{n}\frac{b}{1+b}$. It is clear that x' is a Nash strategy that is quasi strict. Since the payoff matrix of C restricted to the support of x has zeros on the diagonal and $B_{kl} = 1$ if and only if $B_{lk} = b$, B is negative definite and x' is an ESS.

Let $n = 2k$. Consider strategies e_{ij} $(i = 1, \dots, n$ and $j = 1, 2)$ defined as follows. For $i = 1, \dots, k$ and $j = 1, 2$ let e_{ij} denote the strategy: "Send message m_i. Play action j if m_i is received. Play action 1 if message $m \in \{m_{i+1}, \dots, m_{i+k}\}$ is received and play action 2 otherwise." For $i = k+1, \dots, 2k$ and $j = 1, 2$ let e_{ij} denote the strategy: "Send message m_i. Play action j if m_i is received. Play action 1 if message $m \in \{m_{i+1}, \dots, m_{2k}\} \cup \{m_1, \dots, m_{i-k-1}\}$ is received and play action 2 otherwise."

Next we show the following claim: The game restricted to these strategies has a completeley mixed equilibrium y, and this equilibrium is an ESS in the original game. Moreover, the payoff it obtains against itself approaches $(1+b)/2$ as $k \to \infty$.

Proof: It is clear that for any completely mixed equilibrium strategy y we must have (in order to be indifferent between e_{i1} and e_{i2}) that $y_{i1} = by_{i2}$. Let $\alpha_i = y_{i1} + y_{i2}$ and let $e_i = \frac{b}{1+b}e_{i1} + \frac{1}{1+b}e_{i2}$. For $i = 1, \dots, k-1$ indifference between e_i and e_{i+1} is equivalent to

$$\alpha_i\left(\frac{b}{1+b} - 1\right) + \alpha_{i+1}\left(b - \frac{b}{1+b}\right) + \alpha_{i+k+1}(1-b) = 0.$$

Indifference between e_k and e_{k+1} is equivalent to

$$\alpha_k\left(\frac{b}{1+b} - 1\right) + \alpha_{k+1}\left(b - \frac{b}{1+b}\right) = 0.$$

For $i = k+1, \dots, 2k-1$ indifference between e_i and e_{i+1} is equivalent to

$$\alpha_i\left(\frac{b}{1+b} - 1\right) + \alpha_{i+1}\left(b - \frac{b}{1+b}\right) + \alpha_{i-k}(1-b) = 0.$$

Define $\alpha_j = \beta_j / \sum_1^{2k} \beta_j$ where

$$\beta_j = 1 + b^{4k} + (b^2 - 1)b^{4k-4j} \quad \text{for } j = 1, \dots, k,$$

$$\beta_j = 1 + b^{4k} + (1 - b^2)b^{8k+2-4j} \quad \text{for } j = k+1, \dots, 2k.$$

It is straightforward to check that $\alpha_j > 0$ for all $j = 1, \dots, 2k$ and that the indifference conditions are met. Hence, there is a completely mixed strategy y.

This is an ESS in the full game (because of the fact that a matrix with 0 on the diagonal and b and 1 off the diagonal with $B_{kl} = b \Leftrightarrow B_{lk} = 1$ is negative definite).

Note that

$$y \cdot Cy = \frac{b}{1+b}\sum_{i=1}^{2k} \alpha_i^2 + \frac{1+b}{2}\left(1 - \sum_{i=1}^{2k} \alpha_i^2\right).$$

We need to show that our so constructed equilibrium strategy induces an expected payoff close to the efficient one. This is accomplished by showing that

$$\sum_{i=1}^{2k} \alpha_i^2 = \frac{\sum_{i=1}^{2k} \beta_i^2}{(\sum_{i=1}^{2k} \beta_i)^2} \to 0 \quad \text{as } k \to \infty.$$

Note that

$$\beta_{2k} \geq \beta_{2k-1} \geq \cdots \geq \beta_{k+1}, \quad \text{and}$$

$$\beta_1 \geq \beta_2 \geq \cdots \geq \beta_{k-1} \geq \max\{\beta_k, \beta_{2k}\} \geq \beta_{k+1}.$$

Hence,

$$\frac{\sum_{i=1}^{2k} \beta_i^2}{(\sum_{i=1}^{2k} \beta_i^2)^2} \leq \frac{2k(\max \beta_i)^2}{4k^2(\min \beta_i)^2} = \frac{\beta_1^2}{2k\beta_{k+1}^2} \to 0$$

as

$$\frac{\beta_1}{\beta_{k+1}} = \frac{1 + b^{4k} + b^{4k-2} - b^{4k-4}}{1 + b^{4k-2}} \to \frac{b^4 + b^2 - 1}{b^2}. \qquad \square$$

4.2. Voluntary communication before task allocation

Now we include the opportunity for a player not to show up to pre-play communication. Recall that $\hat{x}$ denotes the efficient strategy in case of mandatory cheap talk with n messages. Let $\hat{y}$ denote the strategy that behaves just like $\hat{x}$ except that it plays action 2 when the other player does not show up. Let $\hat{q} = \hat{x} \cdot C\hat{x}$. The efficient payoff is now obtained by the strategy to not show up and to play action 1 with probability $\beta = (1 + b - 2\hat{q})/(2b - 2\hat{q})$ and to show up and to play $\hat{y}$ with probability $1 - \beta$.[9] Of course, this strategy is not an ESS as it is not even Nash. The conditions for ESS are very similar to those obtained under mandatory cheap talk: not all players show up to communication, no-shows choose a pure strategy, during communication all cheap talk messages are used and when sent and received cheap talk message differ players choose different tasks.

Lemma 4. *Let $M^V = \{m^H, m_1, \ldots, m_n\}$ and let $\mathscr{G}_2$ be a task allocation game.*

(i) *Each element of an ES Set of $\mathscr{G}_2^{M^V}$ is an ESS and has each of its best replies in its support.*

[9] Note that $\beta \to 0$ as $n \to \infty$.

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(ii) *Consider two players matched with each other each playing the same ESS x of $\mathcal{G}_2^{M^V}$. Then each message (including m^H) is sent with positive probability. Each time m^H is sent, the same pure action i from $\mathcal{G}_2$ is played. Players showing up to pre-play communication play a best response to action i if their opponent does not show up. x conditional on not sending m^H is an ESS under mandatory communication.*

Proof: Consider behavior after sending or receiving m^H. Consider an element x of an ES Set. Assume that a player sending message m_i and receiving message m^H is indifferent between the two actions. Then drift to playing more of action k after sending m_i and receiving m^H will not change the payoffs and hence there is an element of the ES Set where players play the same action k whenever they do not send m^H but receive m^H.

Consider such an element $\tilde{x}$ of an ES Set and let y be defined as $\tilde{x}$ conditional on not sending m^H. Consider now the reduced form of our game where there are only three strategies $(m^H, 1)$, $(m^H, 2)$ and y. Then y weakly dominates (m^H, k) as $y \cdot Wy > 0$. Consequently, (m^H, k) cannot be contained in the support of x. This means that for any element of an ES Set, m^H is associated with play of a unique action and any player arriving at the pre-play communication will best respond to the anticipated action of m^H choosers.

The rest of the proof is analogous to that of Lemma 2. □

Next we present some results regarding existence and achievable payoffs of ESS. Consider an ESS x of the game with mandatory cheap talk and let $q = x \cdot Cx$ denote the payoff it induces. Let $[x, k]$ denote the strategy "show up and send messages according to x, and react to cheap talk messages as x does; play action k if the other player does not show up." Let Y^k denote the set of pure strategies used with positive probability by $[x, k]$. Let $\neg k = 3 - k$ denote the other action. Consider now the game reduced to the pure strategies in $Y^k \cup \{(m^H, \neg k)\}$ and let B^k denote the corresponding part of the payoff matrix W. This game has a completely mixed symmetric Nash equilibrium y^k unless $q \geq 1$ and $k = 1$; y^k conditional on showing up behaves exactly like $[x, k]$. This can be seen by considering the 2×2 game with only the strategies $(m^H, \neg k)$ and $[x, k]$.

For $k = 1$ this yields the payoff matrix

$$\begin{bmatrix} 0 & 1 \\ b & q \end{bmatrix},$$

which has a symmetric mixed equilibrium whenever $q < 1$; the equilibrium strategy of player one is given by $\left(\frac{1-q}{1+b-q}, \frac{b}{1+b-q}\right)$ and induces $\frac{b}{1+b-q} \in (q, 1)$.

For $k = 2$ the 2×2 game has payoff matrix

$$\begin{bmatrix} 0 & b \\ 1 & q \end{bmatrix},$$

which has a symmetric mixed equilibrium since $q < b$ with equilibrium strategy $\left(\frac{b-q}{1+b-q}, \frac{1}{1+b-q}\right)$ that induces $\frac{b}{1+b-q}$. This payoff is contained in $(q, 1)$ when $q < 1$ and belongs to $(1, q)$ when $q > 1$. Also note that the probability of not showing up is at least $\frac{b-1}{1+b}$ (since $q < (1+b)/2$), independently of the number of cheap talk messages.

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These mixed equilibria of the 2×2 games induce completely mixed equilibria y^k in the game with matrix B^k, and quasi-strict equilibria in $\mathcal{G}_2^{M^V}$, as all best replies are used with positive probability. Since B^k has zeros on the diagonal and b and 1 off the diagonal with $B_{ij}^k = b$ if and only if $B_{ji}^k = 1$, this matrix is negative definite and the quasi-strict equilibrium y^k is in fact an ESS. Since $q \in \left(\frac{b}{1+b}, \frac{1+b}{2}\right)$ the expected payoff induced by y^k is

$$y^k \cdot Wy^k = b/(1+b-q) \in \left(\frac{b+b^2}{1+b+b^2}, \frac{2b}{1+b}\right).$$

To summarize, no-shows obtain a higher payoff than those joining communication but risk being matched against another no-show and thus getting 0. When cheap talk messages are used to achieve a very low payoff (below 1), then adding the option not to show up will increase the payoff though keep it below 1. Thus, making pre-play communication voluntary in task allocation games raises minimal payoffs. On the other hand, when cheap talk messages are used to achieve a higher payoff (above 1), the option not to show up makes things worse. Thus, adding the option of not showing up also raises maximal payoffs if all ESS under mandatory communication induce expected payoffs below one, e.g. when n and b are small such that $n < \frac{b^2+1}{b^2-1}$ (see Lemma 2 (i)). However, voluntary communication is harmful to maximal payoffs when cheap talk messages can be used to induce expected payoffs above one in an ESS. Given Proposition 3 (ii), the latter is possible when there are sufficiently many messages. In this case, the range of expected payoffs attainable in an ESS under mandatory pre-play communication strictly contains that attainable under voluntary pre-play communication.

Proposition 5. *Let $M^V = \{m^H, m_1, \ldots, m_n\}$ and let $\mathcal{G}_2$ be a task allocation game.*

(i) *$\mathcal{G}_2^{M^V}$ has an ESS y that induces an expected payoff strictly below 1.*

(ii) *If $n < \frac{b^2+1}{b^2-1}$ then for each ESS z of $\mathcal{G}_2^M$ there exists an ESS x of $\mathcal{G}_2^{M^V}$ such that $z \cdot Cz < x \cdot Wx < 1$. If n is sufficiently large then there exist ESS's z and z' of $\mathcal{G}_2^M$ such that $z \cdot Cz < x \cdot Wx < z' \cdot Cz'$ holds for any ESS x of $\mathcal{G}_2^{M^V}$.*

(iii) *Expected payoffs induced by an ESS of $\mathcal{G}_2^{M^V}$ are contained in $\left(\frac{b+b^2}{1+b+b^2}, \frac{2b}{1+b}\right)$ where $\frac{b}{1+b} < \frac{b+b^2}{1+b+b^2} < \frac{2b}{1+b} < \frac{1+b}{2}$.*

(iv) *For every $\delta > 0$ there exists n_0 such that if $n > n_0$ then there exists an ESS y' of $\mathcal{G}_2^{M^V}$ that induces an expected payoff above $2b/(1+b) - \delta$.*

Proof:

Part (i). Let x be an ESS of the mandatory cheap talk game with $q := x \cdot Cx < 1$ as constructed in Proposition 3(i). Then y^1 as constructed in the main text is a quasi-strict equilibrium strategy and hence is an ESS. Clearly, y^1 induces an expected payoff less than 1.

Part (ii). This follows from the discussion before the Proposition.

Part (iii). This follows from the fact that any ESS of the mandatory communication game induces an expected payoff between $\frac{b-b^{2n}}{1-b^{2n}}$ and $\left(1-\frac{1}{n}\right)\frac{1+b}{2} + \frac{1}{n}\frac{b}{b+1}$. (See Proposition 3.)

Part (iv). Let x be an ESS of the mandatory cheap talk game with $q := x \cdot Cx > (1+b)/2 - \delta$ as constructed in Proposition 3(ii). Then y^2 as constructed in the main text is a quasi-strict equilibrium strategy and hence is an

ESS. It induces an expected payoff of at least $\frac{2b}{1+b+2\delta}$ which is arbitrarily close to $2b/(1+b)$ provided δ is sufficiently small. □

5. Conclusion

We reveal an implicit assumption of the classic cheap talk models, namely that players are forced to communicate, and consider what happens when this is relaxed. Whether our model of voluntary communication is superior for reaching efficiency depends on the underlying game. In pure coordination games this is true as the inefficient evolutionarily stable equilibria are now ruled out. As it is in the interest of all to play the best action it does not matter whether players communicate or not. The possibility not to participate in the communication breaks the babbling equilibrium in which all messages are sent. In task allocation games we get the opposite result as we find mandatory cheap talk superior. Near efficiency can only be reached if players coordinate and perform each task about half the time. This requires that all players show up most of the time to the communication round. However, players have a natural incentive also not to show up when such an action is rewarded with a payoff higher than the nearly efficient payoff. Because such offers that make all worse off can not be ruled out in these evolutionary models, players have to be exogenously forced to communicate in order to maintain the possibility of reaching outcomes close to the efficient one.

Both for simplicity and for being the traditional approach, we chose the concept of ESS and its set-wise generalization ES Set to select outcomes.[10] Alternatively one might choose to work directly with a selection dynamic such as the replicator dynamic or a more general aggregate monotone dynamic. This would tie our results to boundedly rational "optimal" learning by imitation scenarios as developed by Schlag (1998, 1999). In fact, under an aggregate monotone dynamic any ESS or ES set will be an asymptotically stable strategy and an asymptotically stable set (in the definition where the set is attracting and each point is Lyapunov stable) respectively. While generally asymptotically sets that are not ES Sets may also exist, for the games analyzed in this paper, it can be verified (but is beyond the scope of this paper to show that) that this is not the case.

Other mechanisms for reaching efficiency in coordination games have been suggested. Van Damme and Hurkens (1996) assume that players can consider when to choose an action. Committing to an early choice is similar to committing by not attending pre-play communication and in fact induces efficient outcomes (see their Table 1b). Sobel (1993) considered infinitely repeated games based on two population models where the first few stages of the repeated games are interpreted as cheap messages (see also Balkenborg (1995)). Notice that the option of publicly burning money before choosing actions will not guarantee efficiency when selecting outcomes using ESS (see Ben-Porath and Dekel, 1992[11]).

[10] Recently there has been lots of work with finite population dynamics. However, typically these dynamics do not select only mixed strategies which we find the natural solutions to our task allocation game.
[11] In fact, notice that after identifying L with U and D with R, that $0.75 * ODU + 0.25 * BUD$ is an ESS of the game in Fig 2.3b on page 45 in Ben-Porath and Dekel (1992).

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