Online Appendix B: Optimal Crowdfunding Design

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B Crowdfunding with ex-post sales

B.1 Details for Subsection 6.1

We solve the entrepreneur's problem, restricting ourselves to pure strategy equilibria.[1] Suppose the entrepreneur uses production pivot n and ex-post pricing pivot $m \geq n$ so that $p_2(k) = v_H$ if $k \geq m$ and $p_2(k) = v_L$ otherwise. If the entrepreneur includes L-type funders by setting minimum price $p = v_L$, the maximal bid an H-type is willing to pay, $\tilde{b}^I_{nm}$, is given by the binding IC_H constraint $(v_H - \tilde{b}^I_{nm})\tilde{S}^{N_1-1}_{n-1} = (v_H - v_L)\tilde{S}^{N_1-1}_n$, so that $\tilde{b}^I_{nm} = \tilde{h}_n v_H + (1 - \tilde{h}_n)v_L$. Profit equals

$$\pi^I_{nm} = \sum_{k=n}^{N_1} \bar{f}^{N_1}_k (k\tilde{b}^I_{nm} + (N_1 - k)v_L - C) + \sum_{k=n}^{m-1} \bar{f}^{N_1}_k N_2 v_L + \sum_{k=m}^{N_1} \bar{f}^{N_1}_k \bar{q}^{N_1}_k N_2 v_H$$

Since $\tilde{b}^I_{nm}$ does not depend on m, the profit-maximizing price pivot is given by $m_I = \min(\{k : \bar{q}^{N_1}_k > \hat{q}\} \cup \{N_1 + 1\})$, whether or not the entrepreneur can commit to it.

If the entrepreneur excludes L-type funders by setting minimum price $p > v_L$, the maximal bid an H-type is willing to pay, $\tilde{b}^E_{nm}$, is given by the binding IC_H constraint $(v_H - \tilde{b}^E_{nm})\tilde{S}^{N_1-1}_{n-1} = (v_H - v_L)(\tilde{S}^{N_1-1}_m - \tilde{S}^{N_1-1}_n)$, where $\tilde{S}^{N_1-1}_m - \tilde{S}^{N_1-1}_n$ is the probability that an H-type funder buys ex-post at $p_2 = v_L$ if he waits. Hence,

$$\tilde{b}^E_{nm} = \frac{v_H(\tilde{f}^{N_1-1}_{n-1} + \tilde{S}^{N_1-1}_m) + v_L(\tilde{S}^{N_1-1}_n - \tilde{S}^{N_1-1}_m)}{\tilde{S}^{N_1-1}_{n-1}}$$

Note that an H-type funder who waits, expects an ex-post price, conditional on production, of

$$\mathbb{E}[p_2] = \frac{v_H\tilde{S}^{N_1-1}_m + v_L(\tilde{S}^{N_1-1}_n - \tilde{S}^{N_1-1}_m)}{\tilde{S}^{N_1-1}_n}$$

[1] Mixed strategy equilibria, where H-type funders randomize between funding and buying ex-post, could be relevant if the entrepreneur wants to commit to price v_H, suffers from the durable goods monopoly problem, and N_2 is small. As discussed in online Appendix B.2, restricting the number of units can create credible price commitment, avoiding mixed strategies.

so that $\tilde{b}^E_{nm} = \tilde{h}_n v_H + (1 - \tilde{h}_n)\mathbb{E}[p_2] > \mathbb{E}[p_2]$. The pivotality motive explains again why H-types are willing to pay extra. Profit equals

$$\pi^E_{nm} = \sum_{k=n}^{N_1} \bar{f}^{N_1}_k (k\tilde{b}^E_{nm} - C) + \sum_{k=n}^{m-1} \bar{f}^{N_1}_k (N_2 + N_1 - k)v_L + \sum_{k=m}^{N_1} \bar{f}^{N_1}_k \bar{q}^{N_1}_k N_2 v_H$$

If the entrepreneur cannot commit to m, she must set it to maximize second-period profit, so $m_E = \min(\{k : \bar{q}^{N_1}_k N_2 v_H > (N_2 + N_1 - k)v_L\} \cup \{N_1 + 1\}) \geq m_I$. She then compares $\max_n\{\pi^I_{nm_I}\}$ with $\max_n\{\pi^E_{nm_E}\}$ and chooses accordingly between inclusion and exclusion.

B.2 Capacity constrained crowdfunding

We now assume the entrepreneur cannot commit to an ex-post price, but that she can limit the amount of rewards. We derive the maximal price p_H that an H-type funder is willing to pay for any combination $(n, \bar{n}_H, \bar{n}_L)$, where n is the production pivot and $\bar{n}_J$ is the number of units of type J rewards. This allows us to calculate maximal profits for any triple $(n, \bar{n}_H, \bar{n}_L)$. The entrepreneur then chooses the profit-maximizing triple.

Let k_H and k_L denote the actual number of H and L rewards sold. Note that

$$k_H(k, \bar{n}_H, \bar{n}_L) = \min\{k, \bar{n}_H\}$$
$$k_L(k, \bar{n}_H, \bar{n}_L) = \min\{N_1 - k, \bar{n}_L\}$$

Note that if $k < \bar{n}_H$, the entrepreneur learns $k = k_H$. If $k_L < \bar{n}_L$, the entrepreneur learns that $k = N_1 - k_L$. In particular, if $\bar{n}_H + \bar{n}_L \geq N_1$, the entrepreneur learns k for sure. On the other hand, if $\bar{n}_H + \bar{n}_L < N_1$, $k_H = \bar{n}_H$ and $k_L = \bar{n}_L$, then the entrepreneur does not learn k exactly. That is, for all $\bar{n}_H \leq k \leq N_1 - \bar{n}_L$ the entrepreneur just learns that k is in this interval.

When k is learned exactly, $p_2(k) = v_H$ if $(\bar{q}^{N_1}_k N_2 + \max\{k - \bar{n}_H, 0\})v_H > (N_2 + N_1 - k_L - k_H)v_L$ and $p_2(k) = v_L$ otherwise. When k is not learned exactly, that is when $\bar{n}_H \leq k \leq N_1 - \bar{n}_L$, $p_2(k) = v_H$ if and only if

$$\sum_{j=\bar{n}_H}^{N_1-\bar{n}_L} \bar{f}^{N_1}_j (\bar{q}^{N_1}_j N_2 + j - \bar{n}_H)v_H > \sum_{j=\bar{n}_H}^{N_1-\bar{n}_L} \bar{f}^{N_1}_j (N_2 + N_1 - \bar{n}_H - \bar{n}_L)v_L$$

Let $K_L = \{k \in \mathbb{N} : n \leq k \text{ and } p_2(k) = v_L\}$ and $K_H = \{k \in \mathbb{N} : k \geq n \text{ and } p_2(k) = v_H\}$.

Clearly, the profit maximizing price for L-type rewards is $p_L = v_L$. To find the maximum p_H consistent with IC_H, note that an H-type funder obtains from bidding p_H:

$$U^H_{\widehat{H}} = \; (v_H - p_H)\left[\sum_{k=n-1}^{N_1-1} \tilde{f}_k^{N_1-1} r_H(k+1)\right] + \sum_{k=n-1}^{N_1-1} (v_H - p_2(k+1))\tilde{f}_k^{N_1-1}(1 - r_H(k+1))$$

where $r_H(k) = \min\left\{1, \frac{\bar{n}_H}{k}\right\}$ denotes the probability of receiving the H-reward, conditional on production, when there are k funders bidding for that reward.

If an H-type funder bids v_L for the L-reward, he obtains

$$U^H_{\widehat{L}} = \; (v_H - v_L)\left[\sum_{k=n}^{N_1-1} \tilde{f}_k^{N_1-1} r_L(k)\right] + \sum_{k=n}^{N_1-1} (v_H - p_2(k))\tilde{f}_k^{N_1-1}(1 - r_L(k))$$

where $r_L(k) = \min\left\{1, \frac{\bar{n}_L}{N_1-k}\right\}$ denotes the probability of receiving the L-reward, conditional on production, when there are $N_1 - k$ funders bidding for that reward. Hence, p_H is defined by $U^H_{\widehat{H}} = U^H_{\widehat{L}}$. The expected payoff for the entrepreneur is thus

$$\begin{aligned}\pi(n, \bar{n}_H, \bar{n}_L) = &\sum_{k \in K_L} \bar{f}_k^{N_1}\Big(k_H p_H + (N_2 + N_1 - k_H)v_L - C\Big) \\ &+ \sum_{k \in K_H} \bar{f}_k^{N_1}\Big(k_H p_H + k_L v_L + (\bar{q}_k^N N_2 + \max\{k - \bar{n}_H, 0\})v_H - C\Big)\end{aligned}$$

The entrepreneur chooses the triple $(n, \bar{n}_H, \bar{n}_L)$ that maximizes her profits. We used numerical methods to find the profit-maximizing pivot and capacities for our illustration in Section 6.2.