Microeconomics II

SPRING 2008

1 Aggregate demand (Varian Ch15)

1. Consider a firm characterized by the following technology:

$$Y = 3L^{1/3}K^{-1/3}$$

   (a) Find the returns to scale. Interpret it.

   (b) Find the marginal product of labor and capital.

   (c) Formulate the profit maximization program.

   (d) Suppose $p = w = 2$ and $r = 1$, find the demand of labor and capital and the supply of the firm.

2. Calculate elasticities as functions of p. Example: $D(p) = 30 - 6p$ has elasticity $\varepsilon(p) = -\frac{p}{5-p}$.

   (a) $D(p) = 60 - p$,
   (b) $D(p) = a - bp$,
   (c) $D(p) = 40p^{-2}$,
   (d) $D(p) = Ap^{-b}$,
   (e) $D(p) = (p+3)^{-2}$,
   (f) $D(p) = (p+a)^{-b}$.

3. Assume a market with two consumers having the following demand behaviors:

$$q_1(p) = \begin{cases} 0, & \text{if } p > 20 \\ 100 - 10p, & \text{if } p \leq 10 \end{cases} \quad \text{and } q_2(p) = \begin{cases} 0, & \text{if } p > 20 \\ 80 - 4p, & \text{if } p \leq 20 \end{cases}$$

   (a) Calculate the price elasticity of q_1 when $p = 5$

   (b) Calculate the price elasticity of q_2 when $p = 5$

   (c) Calculate the price elasticity of q_1+q_2 when $p = 5$

(d) Is it true that the elasticity of the aggregated demand is the sum of the elasticities of the individual demands?

4. Calculate elasticities as functions of p, $\varepsilon(p)$, of the following demand functions:

   (a) $q(p) = \begin{cases} 0, & \text{if } p > a/b \\ a - bp, & \text{if } p \leq a/b \end{cases}$ Calculate $\varepsilon(p)$ when $p = a/b$ and when $p = 0$.

   (b) $q(p) = 1/p^{\alpha}$.

5. The demand function of dog breeders for electric dog polishers $q_b = \max\{200 - p, 0\}$, and the demand function of pet owners for electric dog polishers is $q_o = \max\{90 - 4p, 0\}$.

   (a) At price p, what is the price elasticity of dog breeders' demand for electric dog polishers? What is the price elasticity of pet owners' demand?

   (b) At what price is the dog breeders' elasticity equal to 1? At what price is the pet owners' elasticity equal to 1?

   (c) Draw the dog breeders' demand curve, the pet owners' demand curve, and the market demand curve (clearly label your curves).

   (d) Find a nonzero price at which there is positive total demand for dog polishers and at which there is a kink in the demand curve. What is the market demand function for prices below the kink? What is the market demand function for prices above the kink?

   (e) Where on the market demand curve is the price elasticity equal to 1? At what price will the revenue from the sale of electric dog polishers be maximized? If the goal of the sellers is to maximize revenue, will electric dog polishers be sold to breeders only, to pet owners only, or to both?

6. The demand for kitty litter, in pounds, is $\ln D(p) = 1000 - p + \ln m$, where p is the price of kitty litter and m is income.

   (a) What is the price elasticity of demand for kitty litter when $p = 2$ and $m = 500$? When $p = 3$ and $m = 500$? When $p = 4$ and $m = 1500$?

   (b) What is the income elasticity of demand for kitty litter when $p = 2$ and $m = 500$? When $p = 2$ and $m = 1000$? When $p = 3$ and $m = 1500$?

(c) What is the price elasticity of demand when price is p and income is m? The income elasticity of demand?

7. The demand function for football tickets for a typical game at a large midwestern university is $D(p) = 200,000 - 10,000p$. The university has a clever and avaricious athletic director who sets his ticket prices so as to maximize revenue. The university's football stadium holds 100,000 spectators.

a. Write down the inverse demand function.

b. Write expressions for total revenue $R(q)$ and marginal revenue MR as a function of the number of tickets sold.

c. What price will generate the maximum revenue? What quantity will be sold at this price?

d. At this quantity, what is marginal revenue? At this quantity, what is the price elasticity of demand? Will the stadium be full?
A series of winning seasons caused the demand curve for football tickets to shift upward. The new demand function is $q(p) = 300,000 - 10,000p$.

e. What is the new inverse demand function?

f. Write an expression for marginal revenue as a function of output.

g. Ignoring stadium capacity, what price would generate maximum revenue? What quantity would be sold at this price?

h. As you noticed above, the quantity that would maximize total revenue given the new higher demand curve is greater than the capacity of the stadium. Clever though the athletic director is, he cannot sell seats he hasn't got. He notices that his marginal revenue is positive for any number of seats that he sells up to the capacity of the stadium. Therefore, how many tickets at which price should he sell in order to maximize his revenue.

i. When he does this, what is his marginal revenue from selling an extra seat? What is the elasticity of demand for tickets at this price quantity combination?

2 Market Equilibrium (Varian Ch 16 and 14)

2.1 Some Review Questions

1. What is the effect of a subsidy in a market with a horizontal supply curve? With a vertical supply curve?

2. Suppose that the demand curve is vertical while the supply curve slopes upward. If a tax is imposed in this market who ends up paying?

3. Suppose that all consumers view red and blue pencils as perfect substitutes. Suppose that the supply curve for red pencils is upward sloping. Let the price of red pencils and blue pencils be p_r and p_b. What would happen if the government put a tax only on red pencils?

4. The US imports about half of its petroleum needs. Suppose that the rest of the oil producers are willing to supply as much oil as the US wants at a constant price of \$25 a barrel. What would happen to the price of domestic oil if a tax of \$5 a barrel were placed on foreign oil?

5. Suppose the supply curve is vertical. What is the deadweight loss of a tax in this market?

2.2 Market equilibrium I

1. Here are the supply and demand equations for throstles, where p is the price in dollars: $D(p) = 40 - p$, $S(p) = 10 + p$.

   (a) Graph the supply and demand curve of throstles and determine equilibrium price and equilibrium quantity.

   (b) Suppose that the government decides to restrict the industry to selling only 20 throstles. At what price would 20 throstles be demanded? How many throstles would suppliers supply at that price? At what price would the suppliers supply only 20 units?

   (c) The government wants to make sure that only 20 throstles are bought, but it doesn't want the firms in the industry to receive more than the minimum price that it would take to have them supply 20 throstles. One way to do this is for the government to issue 20 ration coupons. Then in order to buy a throstle, a consumer would need to present a ration coupon along with the necessary amount of money to pay for the good. If the ration coupons were freely bought and sold on the open market, what would be the equilibrium price of these coupons?

(d) On the graph representing the above supply and demand, shade in the area that represents the deadweight loss from restricting the supply of throstles to 20. How much is this expressed in dollars?

2. The demand curve for ski lessons is given by $D(p_D) = 100 - 2p_D$ and the supply curve is given by $S(p_S) = 3p_S$.

   (a) What are the equilibrium price and quantity?

   (b) A tax of \$10 per ski lesson is imposed on consumers. Write an equation that relates the price paid by demanders to the price received by suppliers. Write an equation that states that supply equals demand.

   (c) Solve these two equations for the two unknowns p_S and p_D. With the \$10 tax, what is the the equilibrium price p_D paid by consumers and the total number of lessons given?

   (d) A senator from a mountainous state suggests that although ski lesson consumers are rich and deserve to be taxed, ski instructors are poor and deserve a subsidy. He proposes a \$6 subsidy on production while maintaining the \$10 tax on consumption of ski lessons. How do the effects for suppliers or for demanders of this policy compare to the effects of a tax of \$4 per lesson?

3. The demand function for merino ewes is $D(P) = \frac{100}{P}$, and the supply function is $S(P) = P$.

   (a) What are equilibrium price and quantity?

   (b) An ad valorem tax of 300% is imposed on merino ewes so that the price paid by demanders is four times the price received by suppliers. What is the equilibrium price paid by the demanders for merino ewes now? What is the equilibrium price received by the suppliers for merino ewes? What is the equilibrium quantity?

2.3 Understanding the notion of surplus

Here is the table of reservation prices for apartments taken from Chapter 1:

1.

Persons	A	B	C	D	E	F	G	H
Price	40	25	30	35	10	18	15	5

   (a) If the equilibrium rent for an apartment turns out to be \$20, which consumers will get apartments?

(b) If the equilibrium rent for an apartment turns out to be \$20, what is the consumer's (net) surplus generated in this market for person A? For person B?

(c) If the equilibrium rent is \$20, what is the total net consumers' surplus generated in the market?

(d) If the equilibrium rent is \$20, what is the total gross consumers'surplus in the market?

(e) If the rent declines to \$19, how much does the gross surplus increase?

(f) If the rent declines to \$19, how much does the net surplus increase?

2. Quasimodo consumes earplugs and other things. His utility function for earplugs x and money to spend on other goods y is given by $u(x, y) = 100x - \frac{x^2}{2} + y$.

(a) What kind of utility function does Quasimodo have?

(b) What is his inverse demand curve for earplugs?

(c) If the price of earplugs is \$50, how many ear plugs will he consume?

(d) If the price of earplugs is \$80, how many ear plugs will he consume?

(e) Suppose that Quasimodo has $\$4,000$ in total to spend a month. What is his total utility for earplugs and money to spend on other things if the price of earplugs is \$50?

(f) What is his total utility for earplugs and other things if the price of earplugs is \$80?

(g) By how much does his utility decrease when the price changes from \$50 to \$80.

(h) What is the change in (net) consumer's surplus when the price changes from \$50 to \$80?

3. F. Flintstone has quasilinear preferences and his inverse demand function for Brontosaurus Burgers is $P(b) = 30 - 2b$. Mr. Flintstone is currently consuming 10 burgers at a price of 10 dollars.

(a) How much money would he be willing to pay to have this amount rather than no burgers at all? What is his level of (net) consumer's surplus?

(b) The town of Bedrock, the only supplier of Brontosaurus Burgers, decides to raise the price from \$10 a burger to \$14 a burger. What is Mr. Flintstone's change in consumer's surplus?

4. Karl Kapitalist is willing to produce $\frac{p}{2} - 20$ chairs at every price, $p > 40$. At prices below 40, he will produce nothing. If the price of chairs is \$100, Karl will produce 30 chairs. At this price, how much is his producer's surplus?

2.4 Market Equilibrium II

1. The price elasticity of demand for oatmeal is constant and equal to -1. When the price of oatmeal is \$10 per unit, the total amount demanded is 6,000 units

   (a) Write an equation for the demand function.

   (b) If the supply is perfectly inelastic at 5,000 units, what is the equilibrium price?

2. The inverse demand function for bananas is $P_d = 18 - 3Q_d$ and the inverse supply function is $P_s = 6 + Q_s$, where prices are measured in cents.

   (a) If there are no taxes or subsidies, what is the equilibrium quantity? What is the equilibrium market price?

   (b) If a subsidy of 2 cents per pound is paid to banana growers, then in equilibrium it still must be that the quantity demanded equals the quantity supplied. What is the new equilibrium quantity? What is the new equilibrium price received by suppliers? What is the new equilibrium price paid by demanders? How much money is handed out in subsidies?

   (c) Express the change in price as a percentage of the original price. If the cross-elasticity of demand between bananas and apples is $+0.5$, what will happen to the quantity of apples demanded as a consequence of the banana subsidy, if the price of apples stays constant? (State your answer in terms of percentage change.)

3. King Kanuta rules a small tropical island, Nutting Atoll, whose primary crop is coconuts. If the price of coconuts is P, then King Kanuta's subjects will demand $D(P) = 1200 - 100P$ coconuts per week for their own use. The number of coconuts that will be supplied per week by the island's coconut growers is $S(p) = 100P$.

   (a) What will be the equilibrium price and quantity of coconuts?

   (b) One day, King Kanuta decided to tax his subjects in order to collect coconuts for the Royal Larder. The king required that every

subject who consumed a coconut would have to pay a coconut to the king as a tax. Thus, if a subject wanted 5 coconuts for himself, he would have to purchase 10 coconuts and give 5 to the king.

When the price that is received b the sellers is p_S, how much does it cost one of the king's subjects to get an extra coconut for himself?

(c) When the price paid to suppliers is p_S, how many coconuts will the king's subjects demand for their own consumption? (Hint: Express p_D in terms of p_S and substitute into the demand function.)

(d) Since the king consumes a coconut for every coconut consumed by the subjects, the total amount demanded by the king and his subjects is twice the amount demanded by the subjects. Therefore, when the price received by suppliers is p_S, the total number of coconuts demanded per week by Kanuta and his subjects is equal to ?

(e) Solve for the equilibrium value of p_S, the equilibrium total number of coconuts produced, and the equilibrium total number of coconuts consumed by Kanuta's subjects.

(f) King Kanuta's subjects resented paying the extra coconuts to the king, and whispers of revolution spread through the palace. Worried by the hostile atmosphere, the king changed the coconut tax. Now, the shopkeepers who sold the coconuts would be responsible for paying the tax. For every coconut sold to a consumer, the shopkeeper would have to pay one coconut to the king. How many coconuts would be sold to the consumers using this plan. How much does the shopkeepers get per coconut after paying their tax to the king, and how much do the consumers pay now?

3 Monopoly (Varian ch 24)

1. Professor Bong has just written the first textbook in Punk Economics. It is called Up Your Isoquant. Market research suggests that the demand curve for this book will be $Q = 2,000 - 100P$, where P is its price. It will cost $\$1,000$ to set the book in type. This setup cost is necessary before any copies can be printed. In addition to the setup cost, there is a marginal cost of \$4 per book for every book printed. Derive the revenue function for Professor Bong's book and the total cost function of production. Furthermore, find price and quantity that will maximize Professor Bong's profit.

2. Suppose that the demand function for Japanese cars in the United States is such that annual sales of cars (in thousands of cars) will be $250 - 2P$, where P is the price of Japanese cars in thousands of dollars.

   (a) If the supply schedule is horizontal at a price of $\$5,000$ what will be the equilibrium number of Japanese cars sold in the United States? How much money will Americans spend in total on Japanese cars?

   (b) Suppose that in response to pressure from American car manufacturers, the United States imposes an import duty on Japanese cars in such a way that for every car exported to the United States the Japanese manufacturers must pay a tax to the U.S. government of $\$2,000$. How many Japanese automobiles will now be sold in the United States? At what price will they be sold?

   (c) How much revenue will the U.S. government collect with this tariff?

   (d) Suppose that instead of imposing an import duty, the U.S. government persuades the Japanese government to impose "voluntary export restrictions" on their exports of cars to the United States. Suppose that the Japanese agree to restrain their exports by requiring that every car exported to the United States must have an export license. Suppose further that the Japanese government agrees to issue only 236,000 export licenses and sells these licenses to the Japanese firms. If the Japanese firms know the American demand curve and if they know that only 236,000 Japanese cars will be sold in America, what price will they be able to charge in America for their cars?

   (e) How much will a Japanese firm be willing to pay the Japanese government for an export license? (Hint: Think about what it costs to produce a car and how much it can be sold for if you have an export license.)

   (f) How much will be the Japanese government's total revenue from the sale of export licenses?

   (g) How much money will Americans spend on Japanese cars?

   (h) Why might the Japanese "voluntarily" submit to export controls?

3. A monopolist has an inverse demand curve given by $p(y) = 12 - y$ and a cost curve given by $c(y) = y^2$.

   (a) What will be its profit-maximizing level of output?

   (b) Suppose the government decides to put a tax on this monopolist so that for each unit it sells it has to pay the government $\$2$. What will be its output under this form of taxation?

(c) Suppose now that the government puts a lump sum tax of \$10 on the profits of the monopolist. What will be its output?

4. A natural monopoly has a cost function of $c(y) = 44 + 48y$ and faces an inverse demand function $p(y) = 96\text{-}4y$. The government regulates this monopoly imposing average cost pricing while maximizing consumer surplus.

   (a) How much will the monopoly produce?

      i. 11
      ii. 1
      iii. 1 and 11
      iv. 35
      v. 6
      vi. 0
      vii. None of the previous answers is correct.

   (b) How much would the monopoly produce if it were not regulated?

      i. 11
      ii. 1
      iii. 1 and 11
      iv. 35
      v. 6
      vi. 0
      vii. None of the previous answers is correct.

   (c) How do consumer surplus and monopoly's profits change if we switch from a regulated to a non regulated market?

Change in consumer's surplus:	Change in producer's profits:
i. -100	i. 100
ii. -170	ii. 170
iii. 0	iii. 0
iv. none of the previous answer is correct	iv. none of the previous answer is correct

5. In Gomorrah, New Jersey, there is only one newspaper, the Daily Calumny. The demand for the paper depends on the price and the amount of scandal reported. The demand function is $Q = 15S^{\frac{1}{2}}P^{-3}$, where Q is the number of issues sold per day, S is the number of column inches of scandal reported in the paper, and P is the price. Scandals are not a scarce commodity in Gomorrah. However, it takes resources

to write, edit, and print stories of scandal. The cost of reporting S units of scandal is $\$10S$. These costs are independent of the number of papers sold. In addition it costs money to print and deliver the paper. These cost \$0.10 per copy and the cost per unit is independent of the amount of scandal reported in the paper. Therefore the total cost of printing Q copies of the paper with S column inches of scandal is $\$10S + 0.10Q$.

(a) Calculate the price elasticity of demand for the Daily Calumny. Does the price elasticity depend on the amount of scandal reported? Is the price elasticity constant over all prices?

(b) Solve for the profit-maximizing price for the Calumny to charge per newspaper. When the newspaper charges this price, the difference between the price and the marginal cost of printing and delivering each newspaper is ?

(c) If the Daily Calumny charges the profit-maximizing price and prints 100 column inches of scandal how many copies would it sell? (Round to the nearest integer.) Write a general expression for the number of copies sold as a function of S.

(d) Assuming that the paper charges the profit-maximizing price, write an expression for profits as a function of Q and S. Using the solution for $Q(S)$ that you found in the last section, substitute $Q(S)$ for Q to write an expression for profits as a function of S alone.

(e) If the Daily Calumny charges its profit-maximizing price, and prints the profit-maximizing amount of scandal, how many column inches of scandal should it print? How many copies are sold and what is the amount of profit for the Daily Calumny if it maximizes its profits?

6. Ferdinand Sludge has just written a disgusting new book, Orgy in the Piggery. His publisher, Graw McSwill, estimates that the demand for this book in the United States is $Q_1 = 50,000 - 2,000P_1$, where P_1 is the price in the U.S. measured in U.S. dollars. The demand for Sludge's opus in England is $Q_2 = 10,000 - 500P_2$, where P_2 is its price in England measured in U. S. dollars. His publisher has a cost function $C(Q) = \$50,000 + \$2Q$, where Q is the total number of copies of Orgy that it produces.

(a) If McSwill must charge the same price in both countries, how many copies should it sell? What price should it charge to maximize its profits? How much will those profits be?

(b) If McSwill can charge a different price in each country and wants to maximize profits, how many copies should it sell in the United States? What price should it charge in the United States? How many copies should it sell in England? What price should it charge in England? How much will its total profits be?

7. A baseball team's attendance depends on the number of games it wins per season and on the price of its tickets. The demand function it faces is $Q = N(20 - p)$, where Q is the number of tickets (in hundred thousands) sold per year, p is the price per ticket, and N is the fraction of its games that the team wins. The team can increase the number games that it wins by hiring better players. If the team spends C million dollars on players, it will win $0.7 - 1/C$ of its games. Over the relevant range, the marginal cost of selling an extra ticket is zero.

   (a) Write an expression for the firm's profits as a function of ticket price and expenditure on players.

   (b) Find the ticket price that maximizes revenue.

   (c) Find the profit-maximizing expenditure on players and the profit-maximizing fraction of games to win.

4 Monopoly behavior (Varian ch25)

1. The Grand Theater is a movie house in a medium-sized college town. This theater shows unusual films and treats early-arriving movie goers to live organ music and Bugs Bunny cartoons. If the theater is open, the owners have to pay a fixed nightly amount of \$500 for films, ushers, and so on, regardless of how many people come to the movie. For simplicity, assume that if the theater is closed, its costs are zero. The nighty demand for Grand Theater movies by students is $Q_S = 220 - 40P_S$, where Q_S is the number of movie tickets demanded by students at price P_S. The nightly demand for non-student movie-goers is $Q_N = 140 - 20P_N$.

   (a) If the Grand Theater charges a single price, P_T, to every body, then at prices between 0 and \$5.50, the aggregate demand function for movie tickets is $Q_T(P_T) = 360 - 60P_T$. What is the profit-maximizing number of tickets for the Grand Theater to sell if it charges one price to everybody? At what price would this number of tickets be sold? How much profits would the Grand make? How many tickets would be sold to students? To non-students?

(b) Suppose that the cashier can accurately separate the students from the non-students at the door by making students show their school ID cards. Students cannot resell their tickets and non-students do not have access to student ID cards. Then the Grand can increase its profits by charging students and non-students different prices. What price will be charged to students? How many student tickets will be sold? What price will be charged to non-students? How many non-student tickets will be sold? How much profit will the Grand Theater make?

2. The Mall Street Journal is considering offering a new service which will send news articles to readers by email. Their market research indicates that there are two types of potential users, impecunious students and high-level executives. Let x be the number of articles that a user requests per year. The executives have an inverse demand function $P_E(x) = 100 - x$ and the students have an inverse demand function $P_U(x) = 80 - x$. (Prices are measured in cents.) The Journal has a zero marginal cost of sending articles via email. Draw these demand functions.

   (a) Suppose that the Journal can identify which users are students and which are executives. It offers each type of user a different all or nothing deal. A student can either buy access to 80 articles per year or to none at all. What is the maximum price a student will be willing to pay for access to 80 articles? (Hint: Recall the lesson on consumer's surplus and the area under the demand curve.) An executive can either buy access to 100 articles per year or to none at all. What is the maximum price an executive would be willing to pay for access to 100 articles?

   (b) Suppose that the Journal can't tell which users are executives and which are undergraduates. Thus it can't be sure that executives wouldn't buy the student package if they found it to be a better deal for them. In this case, the Journal can still offer two packages, but it will have to let the users self-select the one that is optimal for them. Suppose that it offers two packages: one that allows up to 80 articles per year the other that allows up to 100 articles per year. What's the highest price that the undergraduates will pay for the 80-article subscription?

   (c) What is the total value to the executives of reading 80 articles per year? (Hint: Look at the area under their demand curve and to the right of a vertical line at 80 articles.)

   (d) What is the the maximum price that the Journal can charge for 100 articles per year if it wants executives to prefer this deal to

buying 80 articles a year at the highest price the undergraduates are willing to pay for 80 articles?

3. Bill Barriers, CEO of MightySoft software, is contemplating a new marketing strategy: bundling their best-selling wordprocessor and their spreadsheet together and selling the pair of software products for one price. From the viewpoint of the company, bundling software and selling it at a discounted price has two effects on sales: (1) revenues go up due to to additional sales of the bundle; and (2) revenues go down since there is less of a demand for the individual components of the bundle. The profitability of bundling depends on which of these two effects dominates. Suppose that MightySoft sells the wordprocessor for $200 and the spreadsheet for $250. A marketing survey of 100 people who purchased either of these packages in the last year turned up the following facts:

   1) 20 people bought both.

   2) 40 people bought only the wordprocessor. They would be willing to spend up to $120 more for the spreadsheet.

   3) 40 people bought only the spreadsheet. They would be willing to spend up to $100 more for the wordprocessor. In answering the following questions you may assume the following:

   1) New purchasers of MightySoft products will have the same characteristics as this group.

   2) There is a zero marginal cost to producing extra copies of either software package.

   3) There is a zero marginal cost to creating a bundle.

   (a) Let us assume that MightySoft also offers the products separately as well as bundled. In order to determine how to price the bundle, Bill Barriers asks himself the following questions. In order to sell the bundle to the wordprocessor purchasers, the price would have to be less than ?

   (b) In order to sell the bundle to the spreadsheet users, the price would have to be less than ?

   (c) What would MightySoft's profits be on a group of 100 users if it priced the bundle at $320?

   (d) What would MightySoft's profits be on a group of 100 users if it priced the bundle at $350?

   (e) If MightySoft offers the bundle, what price should it set?

   (f) What would profits be without offering the bundle?

(g) What would be the profits with the bundle?

4. Colonel Tom Barker is about to open his newest amusement park, Elvis World. Elvis World features a number of exciting attractions: you can ride the rapids in the Blue Suede Chutes, climb the Jailhouse Rock and eat dinner in the Heartburn Hotel. Colonel Tom figures that Elvis World will attract 1,000 people per day, and each person will take $x = 50 - 50p$ rides, where p is the price of a ride. Everyone who visits Elvis World is pretty much the same and negative rides are not allowed. The marginal cost of a ride is essentially zero.

   (a) What is each person's inverse demand function for rides?

   (b) If Colonel Tom sets the price to maximize profit, how many rides will be taken per day by a typical visitor?

   (c) What will the price of a ride be?

   (d) What will Colonel Tom's profits be per person?

   (e) What is the Pareto efficient price of a ride?

   (f) If Colonel Tom charged the Pareto efficient price for a ride, how many rides would be purchased?

   (g) How much consumers' surplus would be generated at this price and quantity?

   (h) If Colonel Tom decided to use a two-part tariff, he would set an admission fee of ? and charge a price per ride of ?

5. In a congressional district somewhere in the U.S. West a new representative is being elected. The voters all have one-dimensional political views that can be neatly arrayed on a left-right spectrum. We can define the "location" of a citizen's political views in the following way. The citizen with the most extreme left-wing views is said to be at point 0 and the citizen with the most extreme right-wing views is said to be at point 1. If a citizen has views that are to the right of the views of the fraction x of the state's population, that citizen's views are said to be located at the point x. Candidates for office are forced to publicly state their own political position on the zero-one left-right scale. Voters always vote for the candidate whose stated position is nearest to their own views. (If there is a tie for nearest candidate, voters flip a coin to decide which to vote for.)

   There are two candidates for the congressional seat. Suppose that each candidate cares only about getting as many votes as possible. Is there an equilibrium in which each candidate chooses the best position given the position of the other candidate? If so, describe this equilibrium.

5 Oligopoly (Varian ch 27)

1. Two firms sell the same (homogeneous) good in a market and the strategic variable each one sets is quantity (y). The inverse demand function is given by $p(Y) = 48 - 3Y$, where $Y = y_1 + y_2$ is total output and y_1, y_2 are, respectively, the levels of output firm 1 and 2 produce. Marginal costs to produce the good are constant and equal to 2. There are no fixed costs.

   (a) What are the profit functions of the two firms?

   (b) If firms choose quantities simultaneously, what are the reaction functions of the two firms? Draw them in a graph.

   (c) What are the quantities, prices and profits of the two firms in a Cournot equilibrium?

   (d) Suppose now that firm 1 is the leader in quantities and firm 2 is the follower. The follower knows the quantity chosen by the leader when making his choice. What is the reaction function of the follower? What is the profit function of the leader? (Hint: write down the profit function of the leader in a way that it only depends on its output). What are the quantities, prices and profits of the two firms?

   (e) If the two firms decide to form a cartel (i.e. a collusive agreement) what are the price, quantities and profits?

   (f) If the marginal cost of firm 1 is 4 and that of firm 2 is still 2, what are the quantities, prices and profits of the two firms in a Cournot equilibrium?

2. Consider a market for a homogeneous good with inverse demand function

$$p(Y) = 24 - Y.$$

   Suppose the market has N potential firms, each with the same cost function

$$C(y_i) = y_i^2, i = 1, ..., N,$$

   where y_i is the output of firm i, and $Y = y_1 + y_2 + ... + y_N$ is total output.

   (a) Let $N = 2$ and suppose the two firms engage in Cournot competition. Write the firms' profit functions. Find the equilibrium price, the quantities produced by each firm as well as total output Y. (Hint: Consider a symmetric equilibrium, where $y_1 = y_2 = y$.)

(b) Still with $N = 2$, suppose the two firms form a cartel and maximize joint profits. Compute the equilibrium price, the quantities produced by each firm as well as total output Y. Does firm 1 have an incentive to deviate? Explain.

(c) Do the same as in (a) but with $N = 3$. That is,write down the firms' profit functions, find the Cournot equilibrium price, the equilibrium quantities produced by each firm and total output. (Hint: Consider a symmetric equilibrium, where $y_1 = y_2 = y_3 = y$.)

(d) What is the Cournot equilibrium price if N goes to infinity? What will be total production? Explain your answer.

(e) What is the perfectly competitive price if there are two firms ($N = 2$)? What is the perfectly competitive price if N goes to infinity? (Hint: Consider again the symmetric case.)

3. The inverse market demand curve for bean sprouts is given by $P(Y) = 100 - 2Y$, and the total cost function for any firm in the industry is given by $TC(y) = 4y$.

(a) If the bean-sprout industry were perfectly competitive, what would be the industry output and the industry price?

(b) Suppose that two Cournot firms operated in the market. If the firms were operating at the Cournot equilibrium point, what would be industry output, firm output, and the market price?

(c) If the two firms decided to collude, what would be industry output and market price?

(d) Suppose both of the colluding firms are producing equal amounts of output. If one of the colluding firms assumes that the other firm would not react to a change in industry output, what would happen to a firm's own profits if it increased its output by one unit?

(e) Suppose one firm acts as a Stackleberg leader and the other firm behaves as a follower. Then what is the leader's output, the follower's output, industry output and price?

4. Grinch is the sole owner of a mineral water spring that costlessly burbles forth as much mineral water as Grinch cares to bottle. It costs Grinch \$2 per gallon to bottle this water. The inverse demand curve for Grinch's mineral water is $p = 20 - 0.20q$, where p is the price per gallon and q is the number of gallons sold.

(a) What price does Grinch get per gallon of mineral water if he produces the profit-maximizing quantity? How much profit does he make?

(b) Suppose, now, that Grinch's neighbor, Grubb finds a mineral spring that produces mineral water that is just as good as Grinch's water, but that it costs Grubb \$6 a bottle to get his water out of the ground and bottle it. Total market demand for mineral water remains as before. Suppose that Grinch and Grubb each believe that the other's quantity decision is independent of his own. What is the Cournot equilibrium output for Grubb? What is the price in the Cournot equilibrium?

5. Alex and Anna are the only sellers of kangaroos in Sydney, Australia. Anna chooses her profit-maximizing number of kangaroos to sell, q_1, based on the number of kangaroos that she expects Alex to sell. Alex knows how Anna will react and chooses the number of kangaroos that she herself will sell, q_2, after taking this information into account. The inverse demand function for kangaroos is $P(q_1+q_2) = 2000-2(q_1+q_2)$. It costs \$400 to raise a kangaroo to sell.

   (a) Alex and Anna are Stackelberg competitors. Who is the leader and who is the follower?

   (b) Solve for the Stackelberg equilibrium. How many kangaroos will Alex sell? How many kangaroos will Anna sell? What will the industry price be?

6. Consider an industry with the following structure. There are 50 firms that behave in a competitive manner and have identical cost functions given by $c(y) = \frac{y^2}{2}$. There is one monopolist that has 0 marginal costs. The demand curve for the product is given by $D(p) = 1,000 - 50p$.

   (a) What is the supply curve of one of the competitive firms? The total supply from the competitive sector at price p is $S(p) =$?

   (b) If the monopolist sets a price p, the amount that it can sell is $D_m(p) =$?

   (c) What is the monopolist's profit-maximizing output y_m and profit-maximizing price p?

   (d) How much output will the competitive sector provide at this price? What will be the total amount of output sold in this industry?

7. In a market, there are two firms that compete in prices. The total demand is $Y = 10$, i.e. it does not depend on price. If a firm chooses a

price higher than the other one, the firm with the lowest price gets the whole demand and the other firm gets and produces nothing (suppose it has no fixed costs). If both firms charge the same price they split the demand equally.

(a) If the marginal cost of each firm is 5, what price is each firm going to set in a Bertrand equilibrium? What are the quantities and profits of each firm?

(b) If the marginal cost of firm 1 is 5 and the marginal cost of firm 2 is 7, what prices, quantities and profits do the firms obtain in a Bertrand equilibrium? (Hint: Suppose prices can be charged in cents, i.e., 1.01, 1.02, ..., 9.98, 9.99, 10 are all legitimate prices.)

6 Exchange (Varian ch30)

1. Xavier consumes only two goods, T-shirts (T) and shorts (S). All of his income comes from his endowment in these two goods. He does not always receive them in the proportion he likes to consume them, but he can always buy or sell a T-shirt for 1 dollar and a pair of shorts for 2 dollars. His utility function is $U(T, S) = T \cdot S$, where T represents the quantity of T-shirts he consumes and S the quantity of shorts.

a) Suppose his initial endowment is of 50 T-shirts and 100 shorts. What is his income? How many T-shirts and shorts will he want to consume? Draw his budget set showing the initial endowment, the optimal choice and the indifference curves that pass by these points. What is his *net* demand for T-shirts and shorts?

b) Suppose that the price of shorts drops to 1 dollar and the price of T-shirts stays the same. Compare your results with your answer in (a), what happens to the consumption of T-shirts, does it increase? What happens to the consumption of shorts?

c) Given the endowments from (a), what is Xavier's *gross* demand function for T-shirts and what is his *gross* demand function for shorts as functions of the prices for T-shirts and shorts, p_T and p_S? What are his *net* demand functions?

2. We have a small exchange economy where there are two consumers only, Jane and Ian, and two goods, cake and wine. Jane's initial endowment consists of 3 units of cake and 2 units of wine, while Ian's initial endowment consists of 1 unit of cake and 6 units of wine. Jane and Ian have identical utility functions. Jane's utility function is $U(c_J, w_J) = c_J w_J$, and Ian's utility function is $U(c_I, w_I) = c_I w_I$, where c_J and w_J are the units of cake and wine Jane consumes, and c_I, w_I are the units of cake and wine consumed by Ian.

a) Draw an Edgeworth box that illustrates this situation. Represent cake in the horizontal axis and wine on the vertical one. Measure the goods of Jane starting from the lower, left corner of the box, and those of Ian, starting from the upper, right corner of the

box. (Make sure that the height and the width of the box are equal to the total or joint supply of wine and cake.) Represent the initial endowment in the box and call it ω. On the sides of the box, indicate the amounts of cake and wine corresponding to the initial endowment of the two consumers.

b) Draw in red an indifference curve of Jane that represents the allocations for which her utility is equal to 4. Draw in blue or black an indifference curve of Ian that represents the allocations for which his utility is equal to 6.

c) For each Pareto efficient allocation where both agents consume positive quantities of both goods, the marginal rate of substitution between cake and wine must be the same for Jane and Ian. Write down the equation establishing this condition for each agent.

d) Represent in your graph the geometric locus of all allocations that are Pareto Efficient. (*Hint:* The total consumption of both cake and wine equals Jane and Ian's joint endowment of cake and wine. Use this and (c) to first compute the efficient proportion of consumption between cake and wine.)

e) In this example, for each Pareto efficient allocation where both agents consume positive quantities of both goods, the slope of Jane's and Ian's indifference curves will be .... We know that a competitive equilibrium must be Pareto efficient, we also know that at a competitive equilibrium $p_c/p_w = \ldots$. Fill in the blanks.

f) What is Jane's consumption at the competitive equilibrium? And that of Ian? (*Hint:* you have already determined equilibrium *relative* prices (can fix say the price of wine to equal 1). You know the initial endowment of Jane, therefore you can determine the *value* of her initial endowment (this will be her *income*). Knowing this, you can compute her equilibrium demand function. Finally, given that the sum of Jane and Ian's consumptions have to equal total initial endowment, it should be easy to determine Ian's consumption.)

g) In Jane and Ian's Edgeworth box, draw Jane's budget set and represent the competitive allocation and denote it by W.

3. Ana and Brit, consume compact discs and whiskey. Ana has an initial endowment of 60 discs and 10 bottles of whiskey. Brit has 20 discs and 30 bottles of whiskey. They own nothing else. To Ana, a disc d and a bottle of whiskey w are perfect substitutes, her utility function is $U_A(d, w) = d + w$, where d is the number of discs and w the number of bottles of whiskey she drinks. Brit's preferences are more convex, she has a Cobb–Douglas utility function $U_B(d, w) = d \cdot w$.

a) Represent the preferences and initial endowments of Ana and Brit by drawing in an appropriate Edgeworth box (with discs on the horizontal axis) the initial endowments as well as an indifference curve going through the initial endowment.

b) Indicate *all* the (nonnegative) allocations that can be obtained from the exchange between Ana and Brit. Mark in red all the allocations that make *both* Ana and Brit better off relative to the initial endowment. Is the initial endowment an efficient allocation?

c) For Ana and Brit, write the marginal rates of substitution between discs and wine as functions of d and w.

d) Which conditions should be met at every interior Pareto efficient allocation for this economy (independently from the initial endowments)? Indicate all the Pareto efficient allocations interior to the Edgeworth box.

e) Do you see more Pareto efficient allocations? (*Hint*: Look at the boundary of the Edgeworth box. Notice that these are *not* interior or strictly positive allocations.) Try to find all of them.

7 Production (varian Chapter 31)

1. Thelma and Louise finally got into college. Thelma can write term papers at the rate of 10 pages per hour and solve workbook problems at the rate of 3 per hour. Louise can write term papers at the rate of 6 pages per hour and solve workbook problems at the rate of 2 per hour.

   (a) Which of the two has an *absolute* advantage in solving workbook problems? What about in writing term papers? Which of the two has a *comparative* advantage in solving workbook problems? What about in writing term papers?

   (b) Thelma and Luoise each work 6 hours a day. They decide to work together and to produce a combination of term papers and workbook problems that lies on their joint production possibility frontier. Draw a graph of their individual and of their joint production possibility frontiers; use pages of term papers as the x–axis and problems solved as the y–axis.

   (c) If they decide to produce less than 60 pages of term papers, who will write them? How many pages of term papers can they jointly produce if only one of them writes the term papers?

2. Adam and Eve live in "Paradise" island. They consume apples (A, that they eat) and grapeleaves (G, that they wear). Eve has her own business producing apples (the firm is called Apple and she is the sole shareholder), and Adam has his own business producing grapeleaves (the firm is called Levis-Grape and Adam is the sole shareholder). Both of them can work in both firms. If they spend one hour of their leisure producing apples, each of them produces 4 apples. If they work one hour producing grapeleaves, each of them produces 3 grapeleaves. Therefore, production functions are given by

$$\begin{aligned} f_A(L) &= 4L, \\ f_G(L) &= 3L. \end{aligned}$$

Adam and Eve each have 5 hours of leisure time per day (L) as initial endowment and they posess neither apples nor grapeleaves

$$\begin{aligned} \omega_a &= (L = 5, A = 0, G = 0), \\ \omega_e &= (L = 5, A = 0, G = 0). \end{aligned}$$

Leisure does not enter their utility functions, which are given by

$$\begin{aligned} U_a(G, A) &= 6 + \frac{4}{10} \log G + \frac{6}{10} \log A, \\ U_e(G, A) &= 8 + \log G + \log A. \end{aligned}$$

Suppose leisure time (spent working) is the *numeraire*, i.e. $p_L = 1$.

(a) What type of returns to scale do the production functions of apples and grapeleaves display?

(b) Draw the joint production possibility frontier (in apples–grapeleaves space). What are the slopes?

(c) What are prices for apples and grapeleaves? (*Hint*: Maximize profits of Adam's firm relative to contracted labor and do the same for Eve's firm; make sure they both want to produce positive but not infinite amounts.)

(d) Write down Adam and Eve's budget set (*Hint*: Rememeber that both Adam and Eve are sole shareholders of their own firms.)

(e) What is the amount of labour that Adam and Eve supply? (*Hint*: Notice that leisure does not enter their utility functions and that both of them are endowed with 5 hours of leisure.)

(f) Maximize their utility functions subject to their budget constraints and solve for optimal quantities of apples and grapeleaves demanded.

(g) What is the supply of apples and grapeleaves in equilibrium? Check that the labor market clears.

(h) Is this equilibrium efficient? (*Hint*: Is it possible to improve somebody's welfare, without affecting anybody else's?)

(i) For the level of production computed in (g), compute all the efficient allocations between Adam and Eve. (*Hint*: Notice that Eve's consumption of apples and grapeleaves is equal to total production minus Adam's consumption.)