The Governance of Perpetual Financial Intermediaries

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This version: May 2009
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In this paper we investigate the risk sharing potential of financial intermediaries in an overlapping generations economy with agents who may need to consume before a long term investment opportunity pays off. We find that the allocations that intermediaries can offer are constrained by the temptation of the living to liquidate the institutions’ assets and share the proceeds amongst themselves. We characterize a renegotiation constraint for perpetual financial intermediaries, and show that only institutions that can avoid side trading and the rolling over of deposits, can improve on the market allocation.

*JEL Classification:* G21, D91

*Keywords:* Financial Intermediation, Overlapping Generations,
1. Introduction

One of the main services that financial intermediaries - such as banks, pension funds, and insurance companies - are purported to provide is insurance against liquidity risk. One way these institutions share intergenerational liquidity risk is by holding asset buffers over time. The current paper shows that the risk sharing ability of these perpetual institutions is limited because their living members can unilaterally decide to sell all, or part of, the institution’s assets and reallocate the proceeds amongst themselves, to the detriment of future generations.

This paper extends the existing literature that commences with the seminal papers of Edgeworth (1888), Bryant (1980) and Diamond and Dybvig (1983) that show how financial intermediaries can share risk in economies were production comes with gestation lags and assets are illiquid. Diamond and Dybvig (1983) show that if agents face the risk of having to consume before a long term asset pays off, a bank is able to offer a consumption schedule in which early consumers are \textit{ex post} subsidized by late consumers. Jacklin (1987) and Bhattacharya and Gale (1987) however show that the presence of a market poses a constraint on the risk sharing ability of intermediaries. These papers have been followed up by an extensive literature on bank risk sharing.\footnote{E.g. Haubrich and King (1990), Hellwig (1994) and von Thadden (1997, 1998) provide an additional critique of Diamond Dybvig risk sharing. Wallace (1988), Gorton and Penachi (1990), and Diamond (1997) suggest conditions under which banks can offer superior risk sharing.}

An important strand in this risk sharing literature considers the role of financial intermediaries in overlapping generations (OLG) economies. A common feature of these models is that financial intermediaries hold buffers of liquid and illiquid productive assets to smooth consumption (Allen and Gale, 1997), or to take better advantage of productive technologies (Qi, 1994).

In this paper we argue that buffer holding intermediaries are potentially subject to a renegotiation whereby the living stakeholders make themselves uniformly better off by changing the status quo payout and investment rules. If we take the overlapping
generations concept seriously, and do not rely on altruism or an infinitely lived enforcement mechanism, we need to consider the threat of such renegotiations.

Our analysis uses a model that extends the simple Diamond (1965) OLG economy, where the young are endowed with a consumption surplus and the old face a deficit. The economy features a two-period productive technology that returns $R > 1$ and storage. Agents have Diamond Dybvig (1983) preferences: they live either one or two periods and consume at the end of their life. This model has been studied by Qi (1994), Bhattacharya and Padilla (1998), and Fulghieri and Rovelli (1998), among others. Like them, we analyze under what conditions a financial intermediary can improve on the allocation that obtains in a market economy.

The novel feature of this paper is that we assume that intermediaries are controlled by their living members. To model the intermediaries’ governance structure we assume that depositors periodically decide to discontinue the status quo payoff and investment regime, and adopt a feasible alternative allocation. To do this, we let a randomly chosen depositor suggest an alternative allocation, and assume that it is accepted if it makes all other living depositors better off.

We show that a renegotiation of the intermediary’s status quo risk sharing and investment rule may occur if the intermediary’s payoff schedule is too generous. The reason is that the perpetual asset buffer that supports a generous payoff may be liquidated and sold to non-depositors, so that an even more generous schedule can be offered to the living depositors. Although such a redistribution is an out of equilibrium event, it poses constraint on the set of equilibrium allocations. We characterize a renegotiation constraint that defines a convex set of permissible payment schedules so that no feasible alternative regime that uniformly improves the living depositors’ welfare exists.

We find that the renegotiation constraint depends on the industry structure. If the economy has many intermediaries competing with each other, the constraint is stronger than if there is a single monopolist intermediary. The reason for this is that in the former situation, renegotiating institutions can potentially sell assets to each other. This makes renegotiation attractive at lower asset buffer levels. On the other hand, a liquidating
A monopolist institution can only sell existing capital to newborn generations. Because newborns have lower values for intermediate projects, monopolists can maintain larger asset buffers without being subject to renegotiation by its stakeholders.

Both in the competition and monopoly case, we find the renegotiation constraint to be very restrictive, and not met by several allocations suggested in the incumbent literature. Interestingly, we also find that, for both competition and monopoly, the market allocation is on the boundary of the constraint set.

However, since the market allocation is generally not the welfare maximizing allocation within the constrained set, there is still scope for intermediation. We show that an institution that caters to risk-averse agents can achieve an allocation that is superior to the market allocation if it can avoid side trade and prohibit the rolling over of deposits. Institutions that cater to risk-tolerant agents can improve on the market equilibrium under a weaker condition: they only need to impede the sale (or securitization) of deposits.

In an early paper that analyzes the merits of intermediaries in an OLG Diamond-Dybvig economy, Qi (1994) identifies the problem that anonymous late consumers may mimic early consumers, and roll over their deposits. For sufficiently risk-averse agents, this gives rise to an allocation with $r_2 = r_1^2$, were $r_1$ and $r_2$ denote the consumption of early and late consumers respectively. The scalability of the long term project determines the equilibrium allocation, which may well have $r_1 > \sqrt{R}$. Bhattacharya and Padilla (1996) however point out that if interbank deposits are allowed, Qi’s allocation is not sustainable because intermediaries would open deposits with each other instead of investing in the technology. They then show that a government that taxes late consumers and subsidizes newborns can achieve Qi’s allocation, and even the first best allocation. Fulghieri and

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2 Qi (1994) assumes that the maximum per period investment is unity. This gives rise to $r_1 = \frac{\sqrt{\varepsilon^2 + 4(1 - \varepsilon)R - \varepsilon}}{2(1 - \varepsilon)} > \sqrt{R}$, where $\varepsilon$ is the probability of becoming an early consumer.

3 Bhattacharya and Padilla (1996) consider three different tax-subsidy regimes. To achieve first best, an age dependent tax-subsidy scheme is required.
Rovelli (1998) show that the ability of intermediaries to discriminate on age also enables them to attain the first best allocation.

In contrast, we argue that neither taxation nor age verification is sufficient to ensure the first best allocation, as it is not renegotiation proof. To obtain the allocations suggested in the literature, the economy requires, apart from the power to tax or verify depositor’s ages, an infinitely lived enforcer who acts in the interest of all - present and future - generations. Without an infinitely lived enforcement mechanism, intermediaries’ risk sharing potential is severely constrained, by the temptation of the living generations to renegotiate the risk sharing contract, and ostracize future generations.

Our paper is thus similar in spirit as Prescott and Rios-Rull (2000), who show that, in an economy without capital, the threat looms that the young generations abandon the old, and instead restart the economy. In our model, agents form intermediaries that transport a stock of intermediate projects between generations. Such a capital buffer can potentially improve welfare. However, at the same time it is the source of a natural intragenerational conflict. Because we assume that the intermediary is governed by the living, and future generations have not vote, this conflict results in a constraint on risk sharing.

It is important to note that in our analysis there is no aggregate risk, which implies that all equilibria are deterministic and stationary. Gordon and Varian (1988) and Allen and Gale (1997) investigate the role of perpetual financial intermediaries in an economy where there is aggregate (output) risk. In Gordon and Varian (1988), a government takes the role of the intermediary. The authors show that such a perpetual institution could redistribute stochastic labor income over several generations so as to increase overall welfare. As limitations to such intergenerational smoothing they mention the stochastic nature of intertemporal asset transfers, moral hazard on the part of the workforce, and, as we do, the institution’s governance. In Allen and Gale (1997), there is no labor and moral hazard, while there is a safe asset. They show that a banking system can improve welfare vis-à-vis a market economy by holding a buffer of safe assets, which is depleted whenever the risky output falls short of its expectation, and replenished otherwise. Using a result of Schechtmann (1976), they show that such a system can offer its stakeholders the expected value of the stochastic output in all except a negligible number of periods.
Allen and Gale (1997) also point out that such a buffer holding financial system is fragile, because agents abandon it as soon as it underfunded. Our results suggest that even if agents can be forced to join an underfunded system, smoothing will be hampered due to the temptation of contemporaneous generations to seize any potential surplus.

The remainder of this paper is organized as follows: the next section describes the OLG model. In section 3 we examine the market economy equilibrium to provide a benchmark for the coalition equilibrium, and to analyze the agents’ outside option. In section 4 we describe the coalition, including its governance structure, and analyze the equilibrium. Section 5 presents a discussion and interpretation of our findings. Section 6 concludes. Proofs are in the appendix.

2. The OLG Diamond Dybvig economy

The object of our study is an infinite horizon economy with a boundless sequence of overlapping generations of atomistic agents. A new generation, of size normalized to one, is born on every date. Agents are born with an endowment of one unit of a homogeneous good that can be used for consumption or as an input for production. Agents who enter the economy at date $t$ can be of two types: with probability $\varepsilon$ agents are impatient and live for one period only; with probability $1-\varepsilon$ they are patient and live for two periods. Agents born at $t$ who are impatient consume at $t+1$, while patient ones consume at $t+2$. All agents have expected utility preferences with an instantaneous utility functions $U(\cdot)$ that is increasing, strictly concave and twice-continuously differentiable. We also assume that agents are sufficiently risk averse: the relative risk aversion coefficient of $U()$ is everywhere greater than one.

Agents born at date $t$ learn their type after $t$ but before $t+1$. Types may be verifiable. We assume that the population is large enough so that there is no uncertainty on the aggregate distribution of agents in the population.\footnote{This is usually justified in terms of the law of large numbers. Duffie and Sun (2007) provide a rigorous formulation of independent random matching for a continuum population such that the law of large numbers holds exactly.} Hence, at any date $t$, the population contains $3-\varepsilon$
agents: 1-$\epsilon$ patient agents born at $t$-2, 1 agents born at $t$-1 who know their type, and 1 newborns who do not know their type yet. In between dates there are 2-$\epsilon$ agents: 1 young, and 1-$\epsilon$ old.

The economy is endowed with two technologies to produce goods over time. The first technology, storage, allows agents to costlessly transfer consumption from one period to the next. The second technology, the long term technology allows agents to convert one unit of consumption at date $t$ into $R>1$ units of consumption at $t+2$. This technology cannot be interrupted at $t+1$. In the following we will look for equilibrium allocations $\{C_i^1, C_i^2\}_{k\geq 1}$, where $C_i^t$ denotes the consumption of $i$-year olds at time $t$.

2.1. Production scalability, Pareto efficiency, and equilibrium startability.

An issue that is somewhat ignored in the literature and which becomes of particular importance when studying the Pareto efficient allocation is that of the extent to which the production technology can be scaled. Qi (1994) suggested a maximum periodic investment in the long term technology of unity. Subsequent papers have followed this assumption. We will see however that this assumption has the potential to confound fundamentally different allocations. For this reason we assume that the long term project can be scaled up to a multiple of the size of the population’s aggregate endowment, $X>1$.

Then, the socially optimal allocation is $C_i^1 = C_i^2 = X(R-1)+1$, for all $t$. This allocation is attained by periodically investing the maximum amount $X$ in the long term technology. The periodic consumption can be found by subtracting from the periodic good inflows, $XR+1$ ($XR$ from production and 1 from endowments), the periodic investment outflows $X$.

An additional problem arises if rather than consider stationary equilibria, one were to start the economy from scratch. If the economy has a starting date, the interesting intragenarational allocations (such as social optimal allocation) cannot be obtained immediately. In order to obtain the stationary allocations in an economy with a starting date, early generations need to build an asset buffer to obtain the optimal periodic investment. Although startability is not the central theme of our paper, it will be discussed in a later section.
3. The market economy

In this section we analyze the equilibrium allocation in an economy where agents of different types and generations trade securities that are backed by one period old capital (investments in the production technology). We shall call these securities projects. In the following we let $p_t$ denote the date $t$ price of a project started with one unit of consumption good at $t-1$, and which hence pays $R$ goods at $t+1$. Due to their atomistic nature, agents are price takers.

In the capital market economy agents have access to three investment vehicles: the production technology, projects, and storage. They can invest when born and, if they are patient, on their first birthday. We define the key decision variables as follows: $x_t^0$ is the number of projects bought by a newborn agent at date $t$, $x_t^1$ is the number of projects bought by a patient one-year-old at date $t$. Similarly, $y_t^0$ and $y_t^1$ denote the amount invested in the production technology by newborn and patient one-year-olds respectively, while $z_t^0$ and $z_t^1$ denote the amounts stored by newborn and patient one-year-olds at $t$.

Naturally, all surviving agents choose their investments so as to maximize expected utility. We formulate an agent’s problem recursively, and start the analysis with the problem of a patient one year old. Let $m_t^1 = x_t^0 R + y_t^0 p_t + z_t^0$ denote the wealth, in number of goods, of a one year old agent at date $t$. We further define the value function $V(m_t^1)$ as the maximum expected utility a patient one year old can obtain:

$$V(m_t^1) = \max_{x_t^1, y_t^1, z_t^1} U(x_t^1 R + y_t^1 p_{t+1} + z_t^1)$$

(1)

$$m_t^1 \geq x_t^1 p_t + y_t^1 + z_t^1$$

(2)

5 Throughout this article, superscripts denote generations, and subscripts denote decision, transaction, and consumption dates.
The maximand in (1) is the patient agent’s utility from consumption $U(C_{t+1}^2)$, expression (2) is her budget constraint. Because impatient agents consume on their first birthday, we have that the newborn’s maximization problem is:

$$\max_{x_t^0, y_t^0, z_t} \varepsilon U(x_t^0, p_t, y_t^0, z_t^0) + (1-\varepsilon) V(x_t^0, p_{t+1}, y_t^0, z_t^0)$$  \hspace{1cm} (3)$$

$$1 \geq x_t^0 p_t + y_t^0 + z_t^0$$  \hspace{1cm} (4)$$

The arguments in the $U()$ and $V()$ functions in (3) is a $t$-born agent’s first birthday wealth, $m_t^1$, expression (4) is his/her budget constraint. In equilibrium, all agents maximize expected utility and markets clear. The market clearing condition requires that the aggregate investment on date $t$, denoted $y_t$, equals the aggregate holdings of projects on date $t+1$:

$$y_t = y_t^0 + (1-\varepsilon) y_t^1 = x_{t+1}^0 + (1-\varepsilon) x_{t+1}^1 \forall t$$  \hspace{1cm} (5)$$

We define a capital market equilibrium as follows:

**Definition**: An equilibrium for the market economy is a sequence of prices $\{p_t\}_{t \in Z}$ and investment decisions $\{x_t^0, x_t^1, y_t^0, y_t^1, z_t^0, z_t^1\}_{t \in Z}$ such that (i) for all $t$, every agent maximizes his expected utility and (ii) markets clear.

In the appendix we show that there exist infinite stationary two-periodic equilibria, with the following properties:

**Proposition 1** (market equilibrium):

*In any market equilibrium we have, for all $t \in Z$:*

(i) $p_t \in [1, R], p_{t+1} = \frac{R}{p_t}$

(ii) $C_t^1 = p_t, C_t^2 = R$
(iii) If \( p_t > 1 \) then \( y_t = 1 - \varepsilon \frac{R - p_t}{R - 1} \) and \( z^0_t = z^1_t = 0 \)

Note that there is a continuum of equilibria, all of which have two-periodic prices and allocations. The requirement that \( p_{t+1} = R/p_t \) can be seen as a no arbitrage condition: the one period return on primary investments must be equal to the one period return in the secondary market. In the interior equilibria ( \( p_t \in (1, R) \) ), aggregate investment is determinate, and there is no storage. If we impose that patient one-year olds do not invest in the production technology ( \( \eta^0_t = \tau_y \forall t \) ) then the other asset allocation decisions \( (y^0_t, x^0_t, x^1_t) \) are determinate and strictly positive.

Although the OLG Diamond Dybvig model has been studied in the literature, the inherent two-periodicity has not been documented before.\(^6\) Previous papers that investigate the OLG DD model focus on the one-periodic special equilibrium, where \( p_t = \sqrt{R} \forall t \), which offers all agents an allocation of \( \{C^0_t, C^1_t\} = \{\sqrt{R}, R\} \). Naturally, this non-cyclical allocation is the Pareto optimal one for sufficiently risk averse agents, so that it may be that it is the social attractiveness of the one-periodic equilibrium that lead Bhattacharya and Padilla (1996) and Fulghieri and Rovelli (1998) to disregard the cyclical equilibria.

Notice that the market equilibrium cannot offer the Pareto optimal allocation: the socially optimal allocation requires \( X > 1 \) projects to be started on every date. Because the agents are the only economic actors in the market equilibrium, they cannot be forced to keep such a capital buffer alive.

3.1. The startable market equilibrium

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\(^6\) Bhattacharya, Fulghieri and Rovelli (1998) mention two price equilibriums, the one-periodic one (\( p_t = \sqrt{R} \forall t \)), and the corner equilibrium \( p_t = \{...,1,R,1,\ldots\} \). We show that any price process with \( p_t p_{t-1} = R \) is an equilibrium price process.
If the economy has a starting period \((t = 0)\), the first generation determines which of the above mentioned stationary equilibria is played. Since there is no secondary market at date zero, and the only alternative to investing is storing, the first generation solves:

\[
\max_{y_0} \varepsilon U\left((1-y_0) + y_0 p_1\right) + (1-\varepsilon) U\left(1-y_0 \frac{R}{p_1} + y_0 R\right)
\]  

(3)

Since the first derivative of (3)’s maximand is positive for all \(p_1 > 1\), first generation agents invest their entire endowment in the technology, so that we have:

PROPOSITION 2 (startable market equilibrium)

In the \(\mathbb{Z}^+ \cup \{0\}\) economy we have \(p_{\text{odd}} = 1, p_{\text{even}} = R, y_0 = y_{\text{even}} = 1, y_{\text{odd}} = 1-\varepsilon\).

Proposition 2 shows that even though the agents' concave utility function makes the one-periodic stationary equilibrium the most desirable from an overall welfare perspective, it is the least desirable, most cyclical, equilibrium that obtains.

4. Financial Intermediaries

We now investigate whether a financial intermediary can improve on the capital market allocation. We will consider two distinct settings. First we will consider a monopolist intermediary, such as a social security plan imposed by a government. Then we will consider competition, the situation where there is a large number of financial intermediaries that act as price takers for deposits. Banks or insurance companies are natural examples.

Following the literature, we assume that a financial intermediary offers depositors a demandable debt security in exchange for their endowment. This security can be exchanged for \(r_1\) units of consumption by impatient depositors on the period after making a deposit, or for \(r_2\) units by patient agents after two periods.

In addition, we model the governance of the intermediary. In particular we assume that it is governed by its living members, who periodically decide on the investment and the payout schedule \(\{r_1, r_2\}\). In the following we shall limit our attention to stationary coalitions,
and denote their periodic investment \( y \). A coalition is thus described by a vector \( \{y, r_1, r_2\} \). Given that it is governed by its ex-ante identical depositors, for any stationary investment level, any equilibrium allocation solves:

\[
\max_{\eta, r_2} \varepsilon U(\eta) + (1 - \varepsilon)U(r_2) \tag{4}
\]

We shall limit our analysis to institutions who make promises \( \{r_1, r_2\} \) that are feasible in the short and long term and assume that institutions (if there are more than one), are of constant size.\(^7\) Hence, problem (4) is maximized subject to the internal budget constraint:

\[
\varepsilon r_1 + (1 - \varepsilon) r_2 + y \leq 1 + yR \tag{5}
\]

The left hand side of (5), which will be binding in a stationary equilibrium, gives the period outflows: to impatient and patient depositors, and new investment. The right hand side gives the periodic inflow: from new depositors and from maturing projects. Between periods, the institution holds \( 2y \) projects: \( y \) new projects, that have just been started, and \( y \) mature projects, that are about to pay off.

In addition, any stationary investment level must also satisfy the external budget constraint, which is due to the limited scalability of the production technology:

\[
y \leq X \tag{6}
\]

If we solve (4)-(6) we see that the optimal solution is increasing in \( y \) so that an institution wishing to maximize the welfare of its members will try to reach the highest level of investment consistent with the external constraint (6). This implies that in the intermediated economy the first best allocation could potentially be obtained. However, we need to account for the temptation of the institutions’ living members to liquidate and distribute the institutions’ assets leads to additional constraints on the intermediary’s risk sharing potential.

\(^7\) That is, we rule out Ponzi schemes. The reason that coalition sizes are constant is that they have asset buffers with stationary pay-off vectors which cannot be offered to an increasing number of depositors without diluting the current depositors.
4.1. The governance structure of financial intermediaries

Following the incumbent literature, we treat the intermediary as a withdrawal menu and an asset buffer. In the stationary Diamond Dybvig model, the institution determines the withdrawal rules upon foundation. Because once established the withdrawal rights cannot be renegotiated, the DD-bank can be interpreted as an automatic cash dispenser.\footnote{The cash dispenser interpretation was first suggested by Wallace (1988).}

For an OLG economy, the cash dispenser interpretation is problematic because OLG institutions must not only dispense cash, to current and future generations, but also accept new deposits, and make investments. In the extant literature it is assumed that these actions are predetermined by an exogenous welfare maximizer so as to provide welfare to all the institution's (present and future) depositors.

In practice, such institutions (banks, insurance companies, governmental organizations, pension funds, etc.) are governed by a sequence of stakeholder cohorts, consisting of mortal individuals. Consistent with this reality, we study institutions that are exclusively governed by their living members.

To model the governance of the institution we assume that in between payoff dates, general depositors’ meetings take place, in which every depositor is allowed to vote for motions that propose to change the stationary \( \{r_1, r_2\} \) schedule.

A randomly appointed chairperson is allowed to propose an alternative schedule, \( \{r'_1, r'_2, r''_2\} \), which represent the payoffs in the period immediately following the meeting, (denoted, \( t \)) to the two year olds and to the impatient one year olds, and in period \( t+1 \) to the two year olds.

The alternative proposal is implemented if it obtains unanimous support. We consider the unanimity rule as it ensures that no depositor can be expropriated. Clearly, if it were possible to approve a rule with less than unanimous support, coalitions of depositors could form a voting majority and approve sharing rules that expropriated those depositors.
outside the coalition, which would undermine the existence of intermediaries in the first place.

Unanimity also guarantees that any alternative payoff schedule needs to be feasible to be accepted. This means that it has to be possible for the new proposal to be financed by the dividends of terminated projects and, possibly, the sale of some of the intermediate assets owned by the institution. In the following we let $\xi$ stand for the number of intermediate projects sold by a renegotiating institution, and $p(\xi)$ the price per project obtained.

The reason for analyzing possible renegotiations is that in a stationary equilibrium they cannot occur. That is, perpetual intermediaries cannot offer depositors schedules that give rise to renegotiation. Hence we define a stationary coalition equilibrium as follows:

**DEFINITION:** A stationary intermediary equilibrium is characterized by a vector $(y^*, r_1^*, r_2^*)$ that meets the external budget constraint (6), the internal budget constraint (5) with equality, and the following renegotiation constraint:

\[
\varepsilon U(r_1^*) + (1-\varepsilon)U(r_2^*) \geq \varepsilon U(r_1') + (1-\varepsilon)U(r_2'^{t+1})
\]

and

\[
U(r_2^*) \geq U(r_2')
\]

\[
\forall r_2', r_1', r_2'^{t+1} \text{ that meet the following feasibility constraints:}
\]

\[
(1-\varepsilon)r_2' + \varepsilon r_1' \leq y^* R + \xi p(\xi)
\]

\[
(1-\varepsilon)r_2'^{t+1} \leq (y^* - \xi) R
\]

\[
\forall \xi \in [0, y^*]
\]

Equations (7) and (8) require that is impossible to make all coalition members better off by staging a feasible raid on the institution’s assets. The set of feasible deviations is constrained by the coalition’s existing assets, characterized by $y^*$, and the market outside the coalition. The left hand side of inequality (9) denotes the maximum payout in the period immediately following a renegotiation, for a given number of intermediate
projects sold $\xi$. Inequality (10) gives the maximum payout, to patient two year olds, two periods after the renegotiation.

Notice that our equilibrium definition specifies equilibrium allocations instead of equilibrium strategies. Naturally, the equilibrium strategies that support the equilibrium allocation are that the chairman only suggests proposals that will be accepted and that agents only vote in favor of a proposal if it is feasible and makes them weakly better off.

In the context of our model of governance, this constraint requires that the chairperson should not be able to profit from making the following proposal:

"Next period, all two-year olds receive $r_2$ goods. One-year olds can choose between $\frac{r_2}{R}$ projects or $r_1$ goods. Anything left over is for me."

Clearly, all agents would (weakly) accept the above proposal. Under the proposal, the young will self-select into patient and impatient types at the next date, and hence consume $r_1$ if impatient or $r_2$ if patient, the same as their scheduled consumption in the status quo. Hence, upon a renegotiation of the above kind, the two-year olds would be left with a minimum of $yR - \varepsilon r_1$ goods and $y - (1-\varepsilon)\frac{r_2}{R}$ projects. Since the projects need to be exchanged for consumption goods the amount of intermediate assets that has to be sold is:

$$\xi = y - (1-\varepsilon)\frac{r_2}{R} = \frac{\varepsilon R r_1 + (1-\varepsilon)r_2 - R}{R(R-1)}$$

To arrive at the final term, we eliminate $y$ by using the binding internal budget constraint (5). To meet the renegotiation constraint, the proceeds from selling $\xi$ plus the leftover goods cannot be greater than the scheduled payment under the coalition. Hence, the renegotiation constraint can be written as:

$$\xi \rho(\xi) + yR - \varepsilon r_1 \leq (1-\varepsilon)r_2$$

(12)
The price $p(ξ)$ that an institution can achieve for its intermediate assets depends on market structure for financial intermediaries. The simplest case is that of a competitive economy. In such an economy, the price that an institution can fetch for its intermediate projects is the reservation value for depositors of competing institutions. If however, the coalition is a monopolist, it can only sell intermediate goods to newborns. We will consider this latter case in subsection 4.3. First we consider an economy with many small coalitions.

4.2. The renegotiation constraint in a competitive economy.

In a competitive economy with many coalitions, the hypothetical selling price of a coalition’s intermediate projects is determined by the shadow price for one-year investments for patient depositors of other institutions. The maximum price that these depositors are willing to pay for intermediate projects makes them indifferent between withdrawing early and buying projects of the liquidating institution or staying at their own institution. If they take the former action, their payoff is $R_p$, if they stay with the institution, they consume $r_2$. Hence we find that in an economy with a competitive market for deposits we have

$$p(ξ) = \frac{r_1}{r_2} R \tag{13}$$

Substituting this and (11) into (12), gives, after some algebra:

PROPOSITION 3: (renegotiation-proof intermediation with competition)

If, conditional on unanimous support, coalition members can renegotiate a financial intermediary and sell its assets, the institution’s payoff schedule is limited to a convex permissible set of $(r_1, r_2)$ combinations described by:

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9 The reason that the patient depositors of other institutions are the highest bidders for projects is due to the fact that the equilibrium $\{r_1, r_2\}$ will offer depositors a first period return that is higher than a second period return ($r_1^2 > r_2$). This is due to the assumption that agents have relative risk aversion being greater than unity.
\[ r_2 \leq \frac{1}{2(1-\varepsilon)} \left( R - r_1 + \sqrt{R^2 + r_1^2 + 2Rr_1(1-2\varepsilon) - 4Rr_1^2\varepsilon(1-\varepsilon)} \right) \]  

The one periodic market allocation \( \{ \sqrt{R}, R \} \) is on the frontier of this set.

The key insight of proposition 3 is that the threat or renegotiation poses a serious threat on the buffer holding capacity of intermediaries. Figure 1 illustrates the renegotiation constraint and constraint optimal allocation alongside the one-periodic market allocation and several allocations suggested in the literature, for parameter values \( R = 4, \ \varepsilon = \frac{1}{3}, X = 1.2 \), and CRRA coefficient \( \gamma = 4 \).

--- Figure 1 around here ---

In the figure the external budget constraint, denoted by \( a \), and given by the line segment \( \varepsilon r_1 + (1-\varepsilon)r_2 \leq 1 + X(R-1) \), can be interpreted as a budget constraint for a hypothetical infinitely lived welfare maximizing social planner. Point \( A \) gives the socially optimal allocation.

Curve \( b \) depicts the renegotiation constraint given by proposition 3. On this line we find the market allocation, denoted \( M \), and the renegotiation constrained optimal allocation \( B \). The latter point is computed assuming a constant relative risk aversion coefficient of four.

Notice that in order to achieve \( B \), intermediaries need to be able to avoid that patient agents withdraw after one period and open a new deposit on their first birthday. To avoid such rolling over we need \( r_2 < r_1^2 \), or that allocations lie too the left of line \( c \), which depicts the roll over constraint. Note however that there is another mechanism for arbitrage if \( r_1^2 > r_2 \). Instead of opening a deposit and rolling over, newborns can invest in the technology, and offer their project for sale in case they become impatient. Hence, to improve on the market allocation, both deposit rolling over and side trade need to be avoided. Qi (1994) considered the side trade constraint and argued that allocation \( C \) would be optimal. However, our analysis shows that allocation \( C \) is not renegotiation proof.
Bhattacharya and Padilla (1996) provide another argument why allocation $C$ cannot be obtained in a contestable market: offering $r_2 > R$ would invite competing coalitions to invest with each other instead of in the production technology. Bhattacharya and Padilla then show that $C$ can be attained if there is a government that taxes income (or consumption) and offers proportional investment subsidies to newborns. If the government can offer age-dependent subsidy which is proportional on investment, allocation $D$ can be obtained, and if the subsidy can be conditioned on the optimal investment level, it can enforce the Pareto optimal allocation $A$.

As can be seen from the figure, none of the three government transfer schemes suggested by Bhattacharya and Padilla (1996) are renegotiation proof. As long as the renegotiation constraint $d$ lies to the left of the external budget constraint, a motion that calls for an immediate cancellation of the suggested tax-subsidy schemes would gain the vote of the entire living population, and unravel allocations $A, B$ and $C$. This is because between periods all living agents have already received the subsidy, while only future, unborn generations benefit from subsequent subsidies.

Fulghieri and Rovelli (1998) show that allocation $E$ can be obtained if intermediaries can condition payoffs on the age of the depositor, and side trade is ruled out, while the interbank arbitrage constraint, depicted by $d$, which requires $r_1, r_2 \leq R$, is binding.\(^\text{10}\)

4.3. *A monopoly intermediary.*

A reason for the limited potential for intergenerational risk sharing could be the fact that financial intermediaries operate in competitive markets. We now consider a monopoly intermediary and find that also in this setting a renegotiation constraint applies.

As established above, a key element of the renegotiation proof criterion is the $p(\xi)$ function, which denotes the price that a financial intermediary can obtain when it

\(^{10}\) In their paper allocation $E$ coincides with allocation $A$, because they assumed $X = 1$. In our view, line $d$ does not necessarily pass through $A$. Whether allocation $E$ provides more or less intergenerational welfare than allocation $C$ or $D$ depends on the model's parameters, in particular on the maximum profitable investment $X$.\)
liquidates \( \xi \) intermediate projects to the benefit of its current members. In the monopoly case this price is more complicated to derive than in a competitive market because now the only potential buyers of intermediate products are the newborns. If a large monopolist institution offers projects for sale to newborns, the price per project will depend on the amount of projects offered. The following lemma gives the per project price that a coalition obtains when it sells \( \xi \) projects to newborns.

**LEMMA**

*If \( \xi \) projects are sold to a population of newborns, the clearing price per project is:*

\[
p = \begin{cases} 
\frac{\varepsilon R}{\xi(R-1)+\varepsilon} & \text{if } \xi \leq \varepsilon \\
1 & \text{if } \varepsilon < \xi \leq 1 \\
\frac{1}{\xi} & \text{if } \xi > 1 
\end{cases} \tag{15a}
\]

This price satisfies the natural property that it is decreasing in supply. The derivation of (15) (see appendix) follows from proposition 1 and 2. First it is established that the newborns will not store but spend their entire endowment on investing in the technology and buying projects. Then we derive the equilibrium price and quantities that maximize individual utility and clear markets. We find that if fewer than \( \varepsilon \) projects are offered at \( t = 0 \), the subsequent market equilibrium is two-periodic with \( p_0 = p_2 = \frac{\varepsilon R}{\xi(R-1)+\varepsilon} \) and \( p_1 = p_{i+2i} = \frac{R}{p_0} \) for all \( i \in \mathbb{Z}^+ \). If \( \varepsilon < \xi < 1 \), the subsequent price process follows \( p_{2i} = 1, p_{i+2i} = R \). If \( \xi > 1 \), the post-raid price process is given by \( \left\{ \frac{1}{\xi}, R, 1, R, \ldots \right\} \).

To find the set of stationary allocations that satisfy the renegotiation constraint for a monopolist intermediary, we substitute the price function (16) into (12). After some algebra, the resulting inequality gives a constraint on the schedules \( \{r_t, r_2\} \) that intermediary can offer without exposing itself to a renegotiation:
PROPOSITION 4: (monopolist’s renegotiation constraint)

If, conditional on unanimous support, coalition members can renegotiate a financial intermediary and sell its assets, the institution’s payoff schedule is limited to a convex permissible set of \( \{r_1, r_2\} \) combinations described by:

\[
r_2 \leq \min \left( R + \varepsilon \frac{4R^2 + r_1^2(R - 1)^2 - r_1(1 + R)}{2(1 - \varepsilon)}, \frac{R}{1 - \varepsilon} \left( 1 - \frac{2\varepsilon r_1}{1 + R} \right) \right)
\]

(16)

The one periodic market allocation \( \{\sqrt{R}, R\} \) is on the frontier of this set. For \( r_1 > \sqrt{R} \), the line given by (16) lies above the line given by (14).

Equation (14) shows that the renegotiation constraint is the most restrictive of two decreasing \( r_2(r_1) \) lines in \( \mathbb{R}^2 \) that crosses the y-axis in \( r_2(0) = \frac{R}{1 - \varepsilon} \). It can be established that iff \( \varepsilon < \frac{1}{2} \), the frontier has a kink.

Renegotiation constraint (16) implies that even a monopolist that cannot avoid depositors from rolling over deposits cannot improve on the market allocation \( \{\sqrt{R}, R\} \). However, if deposit rolling over can be avoided, for example by verifying age or identity, a centralized intermediary can offer higher welfare than the market economy, and higher welfare than an economy with competing coalitions. The latter conclusion derives from the fact that the renegotiation constraint (16) lies above renegotiation constraint (14). This is also illustrated in Figure 2, which depicts both renegotiation constraints.

--- Figure 2 around here ---

Figure 2 illustrates the renegotiation constraint for a monopolist intermediary for the same parameter values as Figure 1. Renegotiation constraint (16) is given by line \( e \), and the renegotiation constrained optimal allocation – assuming that the coalition can avoid deposit roll over – by point \( E \). As can be seen, the monopolist intermediary can obtain higher social welfare than the competing coalitions, but the resulting equilibrium allocation still falls significantly short of the Pareto optimal outcome.
4.4. Equilibrium

So far we have only characterized the renegotiation constraints, and the identified the constrained optimal allocations. However, there are more equilibria that can obtain. In this section we narrow down the feasible allocations to equilibrium allocations.

First we observe that allocations to the left of line \( b \) (and \( e \), for the monopolist) cannot be equilibria because the depositors can rearrange the payoffs for the young generation without terminating the institution. This is the case because internal budget constraints have slopes larger than the renegotiation constraints (14) and (16), as is illustrated in Figure 3. Similarly, allocations on \( b \) that lie to the North-West of \( B \) (on \( e \) and above \( F \), for the monopoly case) cannot be equilibria. This is because they are associated with asset buffers \((y)\) than are higher than the optimal allocation \( E \). A coalition with allocation above \( B \) \((F)\) could make all depositors better off by reducing its buffer. To further refine the set of equilibrium allocations we observe that a stationary coalition equilibrium must solve:

\[
\max_{r_1, r_2} e U(r_1) + (1-e)U(r_2)
\]  

subject to internal budget constraint (5) and bank raid constraint (14) or (16).

That is, an equilibrium coalition allocation must maximize the expected utility of the young generation, subject to the budget constraint that comes with the stationary investment level \( y^* \). It can be shown that allocations \( \{r_1, r_2\} \) that solve (17) unconstrained by the renegotiation threat lie on the 45 degree line, so that the stationary equilibrium allocation lies either on the 45 degree line, or on the frontier of the permissible set above the 45 degree line, as in figure 3.

--- Figure 3 around here ---

Figure 3 depicts the set of stationary equilibrium allocations if \( R = 4, \ v = \frac{1}{4} \) and \( \gamma = 4 \), for a coalition that can avoid roll-over arbitrage. The thick bold line identifies the set of equilibrium allocations. The thin downward sloping straight lines are the internal budget constraints associated with coalition buffers \( y \). The set of equilibria is a line segment which
is bounded below by the allocation on the 45 degree line that offers agents the reservation utility they can obtain by playing intragenerational Diamond-Dybvig. Line $f$ gives the budget constraint of the intragenerational DD coalition, and $H$ marks the optimal intragenerational DD allocation for $\gamma = 4$.\(^\text{11}\)

5. Discussion

In the previous section we derived the set of renegotiation proof equilibria in an intermediated economy under a series of abstract assumption. The default way to interpret the object of our study the perpetual institution, is as a deposit taking bank. However, other financial intermediaries fit the description of our institution, probably even better. Insurance mutuals, defined benefit pension plans, social security schemes and endowment funds all hold buffers and have sharing rules that are prone to renegotiations. In this section we discuss how our conclusions apply to a more realistic setting.

5.1 Altruism and renegotiation costs.

In our model we assume that as soon as a small surplus for the incumbent depositors becomes available, they will seize it through renegotiation, even though it hurts the newborns. It can be easily conjectured that if depositors have some degree of altruism, they would resist the temptation to renegotiate so that larger asset buffers, periodic investment and increased payout rules are sustainable. Tabellini (1990, 1991), investigates how altruism and majority voting can sustain government debt, and social security systems. It is important however that agent generations care not only about their child and grandchild generation, but about all their offspring generations. It can be shown that if agents only care about the welfare of their immediate offspring, the renegotiation constraint as given by (14) and (16) are unaltered. In this case, we need to consider the coalition as having, on any date, $3-\varepsilon$ members, and owning $\gamma$ intermediate projects and $R\gamma + 1$ units of consumption goods. The newborns can be made equally good of as under the

\[^{11}\text{In the intragenerational DD equilibrium the coalition stores goods. The } G \text{ allocation maximizes ex ante utility subject to the budget constraint } v_1 + \frac{1}{R}(1-\varepsilon)r_2 = 1. \text{ See Diamond and Dybvig (1983).}\]
status quo by offering them \((1 - \varepsilon) \frac{r_1}{R}\) goods (which they immediately invest), and \(\frac{\varepsilon r_2}{R}\) projects. Impatient one year olds require \(\varepsilon r_1\) goods and two-year olds require \((1 - \varepsilon)r_2\) goods. If these agents are just satisfied, the patient one-year olds have thus
\[1 + R y - (1 - \varepsilon) \frac{r_1}{R} - \varepsilon r_1 - (1 - \varepsilon)r_2 = \ldots = y - (1 - \varepsilon) \frac{r_2}{R} = \xi\] goods and \(y - \frac{\varepsilon r_1}{R}\) projects to satisfy their consumption needs. Clearly the projects turn into cash, and the goods can be invested into \(\xi\) projects, and sold on the next period. Hence, to avoid a renegotiation with the consent of one subsequent generation, we once again need equation (12) to hold.

One way to commit to altruism is to institutionalize ‘charters’ or ‘constitutions’, which are meant to prohibit (future) generations to raid a perpetual institution’s assets. Many financial mutuals stipulate in their charter that in case of a voluntary liquidation, leftover proceeds must go to third party stakeholders, such as charities. Although we believe that such efforts can enable the existence of sizable perpetual buffers, there must certainly be a limit. After all, constitutions and charters can be changed by the living.\(^{12}\) Nevertheless, it is reasonable to assume that there are some dead weight renegotiation costs associated with the self interested liquidation of the asset buffer of perpetual institution. Clearly such costs relax the renegotiation constraint, making more desirable allocations sustainable.

5.2. Aggregate risk

In our model we assumed away aggregate risk. As a consequence, the presented equilibrium is deterministic: once an institution is operating \(\{y, r_1, r_2\}\), it will do so forever. Although the threat of renegotiation constraints the set of allocations, voluntary liquidations will never happen: renegotiation is an out of equilibrium event.

Hence the objective of our intermediary is not intergenerational smoothing. If the payoff to the long term production technology, or the aggregate liquidity needs, is stochastic, an

\(^{12}\) This is even the case for small intermediaries in large economies that recur to society (or ‘the law’) to avoid a self interested liquidation of their perpetual buffer. If an institution stipulates that upon liquidation, proceeds will go to a third party (e.g. a charity), this party automatically becomes a stakeholder. If the alternative is continuation, the third party can surely be convinced to agree with a liquidation in which it will not receive all proceeds (or to reward the intermediary’s immediate managers for liquidating)
opportunity for intergenerational smoothing appears. Solving our model with a stochastic \( \tilde{R} \) instead of a fixed \( R \) is very complicated, and beyond the scope of this paper. Nevertheless, it can be conjectured that market values of intermediary’s asset buffers will be stochastic too. If intermediaries hold on to fixed payoff schedules while attempting to maintain fluctuating asset buffers, a renegotiation is bound to obtain eventually when the buffer swells sufficiently, even in the presence of finite dead weight renegotiation costs. Clearly, when such a renegotiation eventually obtains, it leads to enhanced payoff rules for the living generations. Because such increased payoff rules are unlikely to be sustainable, they are bound to eventually cause involuntary liquidation or even crises in bad times.\(^{13}\)

5.3. Side trade constraints.

Apart from the renegotiation constraint, we corroborate that the side trade constraint (Jacklin, 1987) hinders optimal risk sharing. If agents are risk averse, the social optimal stipulates a wealth transfer from late to early consumers. However, if agents are allowed to roll over their deposits, or circumvent the intermediary through side trading (by investing in the long term technology and offering it for sale to depositors in case of becoming impatient) the best feasible allocation is the allocation achieved in the market economy. A natural constraint to side trading is adverse selection: We argue (as does Freeman (1988)), that, unlike perpetual reputation bearing institutions, mortal illiquid agents will find the selling of secondary projects (and hence side trading) costly.

The rolling over of deposits can be prevented too. Fulghieri and Rovelli (1998) suggest that intermediaries only offer deposits to newborns. Another way to prevent the rolling over of deposits is to deny repeat customers, or to require withdrawing depositors to consume their payoffs. Or better, still, through type verification. Age, consumption, and redeposit restrictions are indeed imposed by social security plans and pension funds. Insurance companies engage in type verification.

\(^{13}\) A case in point is the recent failure of Equitable Life, the world's oldest life insurance firm, and the pension fund crisis that is currently plaguing many industrialized economies. These are often attributed to excessive generosity during good times. See for example, "Equitable Life: The blame game" (The Economist, April 14th, 2005, p. 70).
6. Summary and conclusion

In this article we re-examine risk sharing in overlapping generations economies. Such risk sharing obtains through intergenerational trade or through financial intermediaries. Incumbent models in the literature show how financial intermediaries can improve on the market economy by accumulating buffers to smooth consumption or exploit productive technologies. We argue that such buffers may tempt the institution’s contemporary stakeholders to renegotiate the payoffs, to the detriment of successive generations.

We show that if stationary schedules can substituted by a schedule that improves the welfare of all living agents, perpetual financial intermediaries are severely constrained in the allocations they can offer. We characterize a renegotiation constraint, and find that most intergenerational coalition allocations suggested in the literature do not satisfy it. Moreover, we find that the one-periodic market equilibrium is barely permissible if perpetual institutions are controlled by the living generations only.

Institutions that cater to risk averse agents are also constrained by the ability of late consumers to mimic early consumers, and withdraw their claim early, either for redepositing or purchasing secondary assets in the market. Institutions that cannot prohibit this behavior cannot improve on the market equilibrium.
Appendix A: Proofs of propositions and lemma

PROOF OF PROPOSITION 1

Clearly, in the suggested set of equilibria all agents maximize expected utility, by (1) and (2). These are the only equilibria because iff \( p_t p_{t+1} > (\leq) R \), solving (1) gives \( y_t = y_{t-1} = 1 \) (\( y_t = y_{t-1} = 0 \)), in which case (2) leads to \( p_t = 0 \) (\( \infty \)), a contradiction.

To find the periodic investment process, replace \( \frac{R}{p_{t-1}} \) by \( p_t \) in the market clearing condition (2), and solve for \( y_t \). We find:

\[
p_t = \frac{(1 - y_t) + (1 - \varepsilon)(1 - y_{t-1}) p_t}{\varepsilon y_{t-1}} \iff y_t = 1 + (1 - \varepsilon - y_{t-1}) p_t
\]  

(A1)

Because aggregate investment has to be two-periodic too, we have

\[
y_{t-1} = 1 + (1 - \varepsilon - y_{t-2}) p_{t-1} = 1 + (1 - \varepsilon - y_t) p_{t-1}
\]  

(A2)

Substitute the rhs of (A2) into the rhs of (A1), then replace \( p, p_{t-1} \) with \( R \), and rewrite:

\[
y_t = 1 - \varepsilon p_t - (1 - \varepsilon)R + y_t R
\]  

(A3)

From which the investment process given in proposition 1 follows immediately. Q.E.D.

The equilibrium price process \{1,R,1,R\} can be supported by many investment and storage schedules \{\( y_t^0, y_t^1, z_t^0, z_t^1, y_{t+1}^0, y_{t+1}^1, z_{t+1}^0, z_{t+1}^1 \). In this equilibrium not even aggregate investment or storage is defined. The reason for this multiplicity is that for the unlucky generations, the return on projects and investment equals the return on storage. Below are two (of many) schedules that support the most cyclical price process.
### PROOF OF PROPOSITION 2

The first order condition for the first generation's maximization problem (3) is:

\[
\varepsilon \left( p_1 - 1 \right) U' \left( y_0 p_1 + (1 - y_0) \right) + \left( 1 - \varepsilon \right) \left( R - \frac{R}{p_1} \right) U' \left( y_0 R + \frac{(1 - y_0) R}{p_1} \right)
\]  \( \text{(A4)} \)

It is positive for all \( p_1 > 1 \). Hence the first generation invests \( y_0 = 1 \). The equilibrium price and investment process follows from proposition 1.  \( Q.E.D. \)

### PROOF OF LEMMA
First we prove that the first generation agents will not store but spend their entire endowment on buying the \( x \) projects and on investing in the technology. Denote \( z_0 \) the amount stored, and as before, denote \( y_0 \) the amount invested in the technology. The first generation solves:

\[
\max_{y_0} \mathcal{E} U \left( z_0 + y_0 p_t + \left(1 - z_0 - y_0 \right) \frac{R}{p_0} \right) \frac{p_t}{p_0} + \left(1 - \mathcal{E} \right) U \left( y_0 R + \left(1 - z_0 - y_0 \right) \frac{R}{p_0} + z_0 \right) \frac{p_t}{p_1} \quad (A5)
\]

Of which the first derivative with respect to \( z_0 \) is:

\[
\frac{\partial \mathcal{E} U}{\partial z_0} = \mathcal{E} \left(1 - \frac{R}{p_0} \right) U'(\cdot) + (1 - \mathcal{E}) \frac{R}{p_1} \left(1 - \frac{R}{p_0} \right) U'(\cdot) \quad (A6)
\]

Which is negative for all \( p_0 < R \). This proves that the first generation does not store.

To find the clearing price in a liquidating sale, we first consider an interior solution in which all agents spend their entire endowment on projects and on investing. In such an equilibrium, \( y_t \) solves (1) for all \( t \), so that \( p_{odd} = \frac{R}{p_0} \) and \( p_{even} = p_0 \).

The market clearing conditions are:

\[
p_0 = \frac{1 - y_o}{x} \quad (A7)
\]

\[
p_t = \frac{\left(1 - y_t \right) + \left(1 - \mathcal{E} \right) \left(1 - y_{t-1} \right) \frac{R}{p_{t-1}}}{\mathcal{E} y_t} \quad \forall t > 0 \quad (A8)
\]

Equation (A8) implies that \( y_t \) is two-periodic. From proposition 1 we know that:

\[
y_t = 1 - \mathcal{E} \frac{R - p_t}{R - 1} \quad \forall t > 0. \quad (A9)
\]

From equation (A8) it also follows that \( y_0 = y_{even} \) so that:

\[
y_0 = 1 - \mathcal{E} \frac{R - p_0}{R - 1}. \quad (A10)
\]

29
Substitute (A10) into (A7) to find expression (16a). If \( x > \varepsilon \), (16a) results in \( p_0 < 1 \), and by proposition 1, to a \( p_i > R \). This in turn implies (from (A9)) that \( y_i > 1 \), contradicting our assumption of an interior solution.

Trivially, if more than \( \varepsilon \) projects are offered, the price they fetch will not be less than unity ((16b) of lemma), unless more than one project is offered. In the latter case all the goods of the first generation goes to buying projects, so that the price per project is \( \frac{1}{x} \) ((16c) of lemma). Q.E.D.

PROOF OF PROPOSITION 3

We look for a function \( r_2(r_i | \varepsilon, R) \) so that (15) holds with equality. This expression is the frontier of the permissible set. If (16a) describes the price equation, we can find the frontier by substituting (16a) in (15):

\[
\frac{(\varepsilon r_i + (1-\varepsilon)r_2 - 1)}{R-1} - (1-\varepsilon)\frac{r_2}{R} \varepsilon R + \frac{\varepsilon r_i + (1-\varepsilon)r_2 - 1}{R-1} R - \varepsilon r_i - (1-\varepsilon)r_2 = 0
\] (A11)

Which can be written as a quadratic equation in \( r_2 \), of which the positive root is:

\[
\frac{r_2}{R} = R + \varepsilon \sqrt{4R^2 + \varepsilon^2 (R-1)^2 - \varepsilon (1+R)}
\] (A12)

This gives us the first part of (17). If (16b) is the relevant price equation, we need to substitute this into (15). We obtain:

\[
\frac{(\varepsilon r_i + (1-\varepsilon)r_2 - 1)}{(R-1)} - (1-\varepsilon)\frac{r_2}{R} + \frac{(\varepsilon r_i + (1-\varepsilon)r_2 - 1)}{(R-1)} R - \varepsilon r_i = (1-\varepsilon)r_2
\] (A13)

Which, after some algebra, becomes:
The second part of (17). Finally, if (16c) is the relevant price equation, we get for (15):

\[ 1 + \frac{(\varepsilon r_i + (1-\varepsilon)r_2 - 1)}{(R-1)} R - \varepsilon r_i \leq (1-\varepsilon)r_2 \]  

Which reduces to:

\[ r_2 \leq \frac{1 - \varepsilon r_i}{1 - \varepsilon} \]  

Observe that the price-equation ((16a)-(16c)) can be written as:

\[ p(x) = p_0(x) = \min \left( \max \left( \frac{\varepsilon R}{x(R-1)+\varepsilon}, 1 \right), \frac{1}{x} \right) \]

Because the number of projects sold in a raid is 

\[ y - (1-\varepsilon) \frac{r_2}{R} = \ldots = \frac{R \varepsilon r_i + (1-\varepsilon)r_2 - R}{R(R-1)} \]

which increases in both \( r_i \) and \( r_2 \), we may combine (A12), (A14) and (A16) to describe the permissible as follows:

\[ r_2 \leq \max \left( \min \left( \frac{R}{1 - \varepsilon} \right), \frac{2\varepsilon R}{(1 - \varepsilon)(1 + R)} \right) \left( 1 - \varepsilon r_i \right) \frac{1}{1 - \varepsilon} \]  

Because the minimum of the first and second term is always greater than the third term (in the relevant region \( r_i, r_2 > 0 \)) we can omit the outer max-operator. The proposition that \( \left\{ \sqrt{R}, R \right\} \) lies on the frontier can be proven by substitution.  

\[ Q.E.D. \]

PROOF OF PROPOSITION 4

We prove proposition 4 in four steps. We first show that the problem (21)-(22), if unconstrained by bank raid constraint (17), has a solution with \( r_1 = r_2 \). Second we prove that the crossing point of the first and second component of (17) lies above the 45 degree
line. Third, we prove that the slope of the first component of (17) is greater than the slope of budget constraint (22), and finally we prove uniqueness of point $E$.

Due to non-satiability (22) is binding, so that we rewrite (21) as:

$$\max_{r_1} \varepsilon U(r_1) + (1-\varepsilon)U\left(\frac{1+y^*(R-1)}{1-\varepsilon} - \frac{\varepsilon}{1-\varepsilon} r_1\right)$$  \hspace{1cm} (A19)

The first order condition of which is:

$$\varepsilon U'(r_1) - \varepsilon U\left(\frac{1+y^*(R-1)}{1-\varepsilon} - \frac{\varepsilon}{1-\varepsilon} r_1\right) = 0$$  \hspace{1cm} (A20)

proving that $r_1 = r_2$.

For the second part of the proof, we observe that the second component of (17),

$$r_2 = \frac{R}{1-\varepsilon} - \frac{2\varepsilon R}{(1-\varepsilon)(1+R)} r_1,$$

crosses the 45 degree line at $r_1 = \frac{R(R+1)}{1-\varepsilon + R + \varepsilon R}$. We need to show that this is greater than first component of (17),

$$R + \varepsilon \sqrt{4R^2 + r_1^2 (R-1)^2} - r_1 (1+R)$$

evaluated at $r_1 = \frac{R(R+1)}{1-\varepsilon + R + \varepsilon R}$. Or, we need to show that:

$$\frac{R(1+R)}{1-\varepsilon + R + \varepsilon R} \geq \frac{R + \varepsilon \sqrt{4R^2 + \frac{R^2 (1+R)^2}{(1-\varepsilon + R + \varepsilon R)^2} - \frac{R(1+R)^2}{1-\varepsilon + R + \varepsilon R}}}{2(1-\varepsilon)}$$  \hspace{1cm} (A21)

Multiplying both sides by $1-\varepsilon + R + \varepsilon R$, dividing by $R$, canceling terms, then dividing both sides by $\varepsilon$, and multiplying sides by $2(1-\varepsilon)$ gives:

$$(1+R)^2 - 2(R-1)(1-\varepsilon) \geq \sqrt{4(1-\varepsilon + R + \varepsilon R)^2 + (1+R)^2 (R-1)^2}$$  \hspace{1cm} (A22)

And hence

$$\frac{4(1-\varepsilon + R + \varepsilon R)^2 + (1+R)^2 (R-1)^2 - (1+R)^2 (R-1)^2}{(1+R)^2 - 2(R-1)(1-\varepsilon)} \leq 0$$  \hspace{1cm} (A23)

Collecting terms eventually gives:
\[-4\varepsilon(R-1)^3 - 4(R-1)^2 \leq 0 \quad (A24)\]

Which clearly holds for all $\varepsilon \in [0,1]$ and $R \geq 1$.

The slope of the first component of (17), is

\[
\frac{\partial}{\partial r_i} \left( R + \varepsilon \sqrt{4R^2 + r_i^2(R-1)^2} - r_i(1+R) \right) = \frac{\varepsilon}{2(1-\varepsilon)} \left( \frac{r_i(R-1)^2}{\sqrt{4R^2 + r_i^2(R-1)^2}} - (1+R) \right) \quad (A25)\]

For a given $y'$, the budget constraint (22) can be written as

\[
r_2 = \frac{1 + y'(R-1) - \varepsilon r_i}{(1-\varepsilon)} \quad (A26)\]

The slope of which is $\frac{-\varepsilon}{1-\varepsilon} r_i$.

To prove the third step we thus need to show that:

\[
r_i \geq \frac{1}{2} \left( \frac{r_i(R-1)^2}{\sqrt{4R^2 + r_i^2(R-1)^2}} - (1+R) \right) \quad (A27)\]

or:

\[
(2r_i + 1 + R) \sqrt{4R^2 + r_i^2(R-1)^2} \geq r_i(R-1)^2 \quad (A28)\]

After squaring both sides and simplifying this becomes:

\[4 \left( (R-1)^2 \left( r_i^4 + (R+1)r_i^2 \right) + R(R+1) \left( r_i^2 + R \right) \right) \geq 0 \quad (A29)\]

Which holds for all $r_i \geq 0$ and $R \geq 1$, because all terms of the left hand side are positive.

To prove that point $E$ is unique we only need to establish that the second order derivative of the first component of (17) is positive in the relevant range. We find:

\[
\frac{\partial^2}{\partial r_i^2} \left( R + \varepsilon \sqrt{4R^2 + r_i^2(R-1)^2} - r_i(1+R) \right) = \frac{\varepsilon(R-1)^2}{2(1-\varepsilon)} \left( \frac{4R^2 + r_i^2(R-1)^2 - \frac{1}{4} r_i}{(4R^2 + r_i^2(R-1)^2)\sqrt{4R^2 + r_i^2(R-1)^2}} \right) \quad (A30)\]
Which is clearly positive over the relevant range. \( Q.E.D. \)

PROOF OF PROPOSITION 5

Uniqueness of the equilibrium follows from the fact that the maximized expected utility of the first generation decreases in \( y^* \) while the maximized expected utility of the stationary generation increases in \( y^* \). This proves that there is a \( y^* \) where the maximized expected utilities are equal. As long as the point \( H \) stays on the diagonal, risk aversion does not affect the utility of the stationary generation, but decreases the utility of the first generation. Hence we find that with decreasing risk aversion, the point \( H \) moves upward along the black line in Figure 2. \( Q.E.D. \)
References


Figure 1

Constraints and coalition allocations under different assumptions in an OLG economy with $\epsilon = \frac{1}{3}$, $R = 4$, and CRRA risk aversion parameter $\gamma = 4$. Line $a$ depicts the Golden Rule constraint, assuming that the maximum investment for a return of $R$ is $X = 1.2$, and point $A$, the Pareto Optimal allocation. Curve $b$ represents the renegotiation constraint in a competitive coalition economy, and point $B$ the renegotiation constrained optimal allocation. Line $c$ represents the rollover constraint and point $C$ the allocation suggested by Qi (1994). Points $D$ (Bhattacharya and Padilla, 1996), is the allocation that can be obtained if an infinitely lived government imposes a tax and subsidy scheme where subsidies are only for the young and proportional to investment. Lines $d$ denote the interbank arbitrage constraint, and point $E$ (Fulghieri and Rovelli, 1998), the allocation that can be obtained by an infinitely lived coalition that can verify agents’ ages or types. Point $M$ is the one-periodic market allocation. The thin convex curves through points $A$ and $B$ are utility indifference curves.
Figure 2

Constraints and coalition allocations under different assumptions in an OLG economy with \( \varepsilon = \frac{1}{3}, \ R = 4, \ X = 1.2 \) and CRRA risk aversion parameter \( \gamma = 4 \). Curve \( b \) represents the renegotiation constraint in a competitive coalition economy. Curve \( e \) represents the renegotiation constraint for a monopolist and point \( F \) the constraint optimal allocation in such an economy.
Equilibrium allocations of the stationary coalition game if $\varepsilon = \frac{1}{3}$, $R = 4$, and CRRA risk aversion parameter $\gamma = 4$, for the competitive (panel A) and monopolist (panel B) coalition economies. The thick black line gives coalition equilibria. The $r_1, r_2$ combinations maximize utility for a given inherited asset buffer. The thin downward sloping lines are the internal budget constraints associated with different asset buffers. Line $f$ is the budget constraint for the intragenerational coalition. $G$ denotes the stationary allocation that offers depositors equal utility as the intragenerational allocation $H$. 

Figure 3
Comments:

- we cannot follow your proposed “theory of the firm” line of argument because it could be argued that our case is one of incomplete contracting (the impossibility of current and future generations to credibly contract with each other). We could battle such arguments but it would detract from the paper. Let’s try to stick to the renegotiation proofness

- this brings me to a second point: renegotiation-proof is a well-established concept in game theory. It’s Standard meaning is not the one we are using so we should also avoid the phrase (lit review Bolton 90EuropeanER)

- Main vocabulary

  o Financial institution, institutions

  o Constraint: renegotiation constraint, deposit increase, SGFI constraint, bounty

  o Renegotiation: sharing, increase deposits, …

  o Assets: capital